HHT Basics and Applications

For Speech, Machine Health Monitoring, and Bio-Medical Data Analysis

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Available Data Analysis Methods for Nonstationary (but Linear) time series

- **Various probability distributions**
- **Spectral analysis and Spectrogram**
- **Wavelet Analysis**
- **Wigner-Ville Distributions**
- **Empirical Orthogonal Functions aka Singular Spectral Analysis**
- **Moving means**
- **Successive differentiations**

Available Data Analysis Methods for Nonlinear (but Stationary and Deterministic) time series

- **Phase space method**
  - **Delay reconstruction and embedding**
  - **Poincaré surface of section**
  - **Self-similarity, attractor geometry & fractals**
- **Nonlinear Prediction**
- **Lyapunov Exponents for stability**

HHT, for Nonstationary, Nonlinear and Stochastic data, consists of the following components:

The Empirical Mode Decomposition:

To generate the adaptive basis, the Intrinsic Mode Functions (IMF), from the data

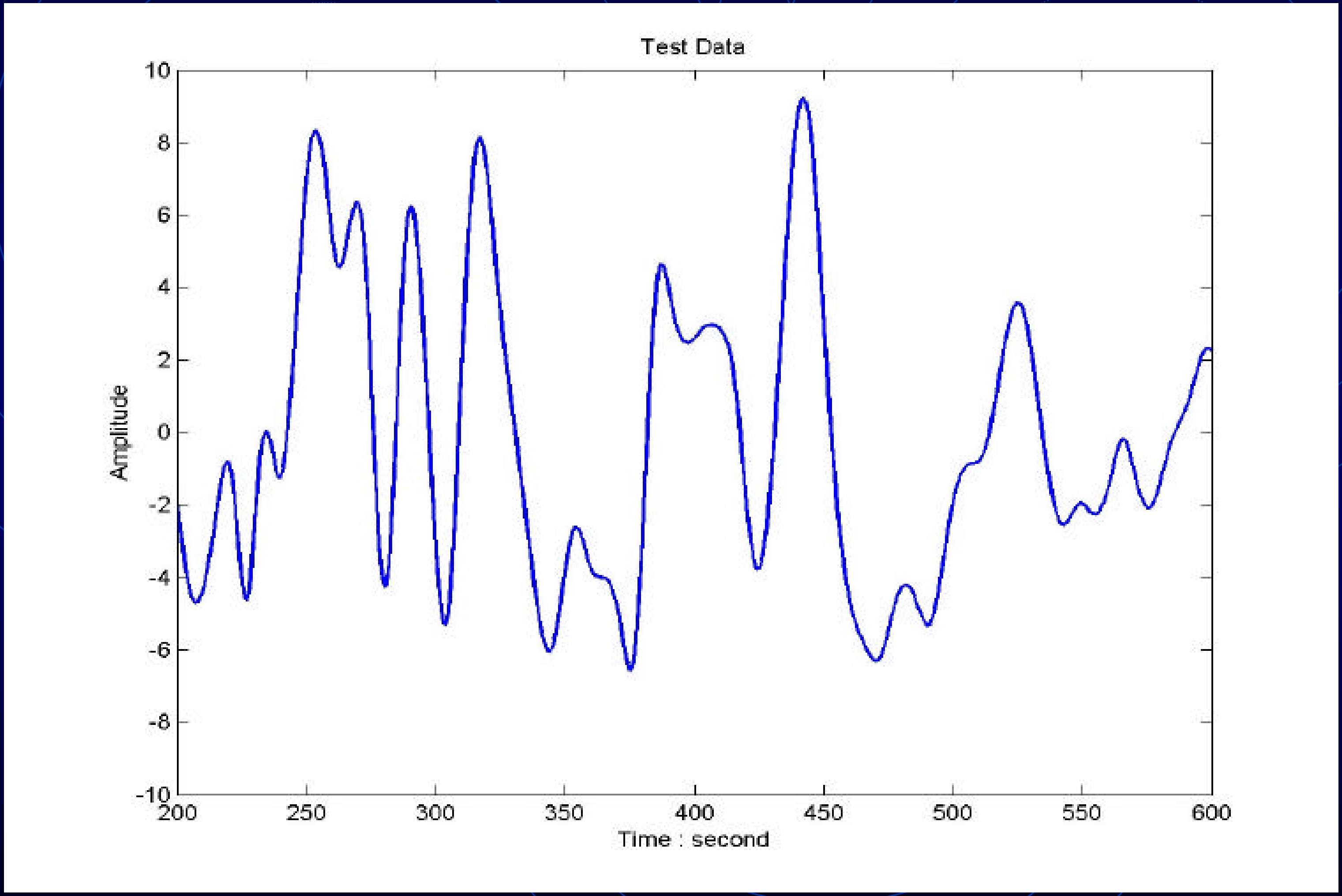
The Hilbert Spectral Analysis:

To generate a time-frequency-energy representation of the data Based on the IMFs

The Empirical Mode Decomposition Method

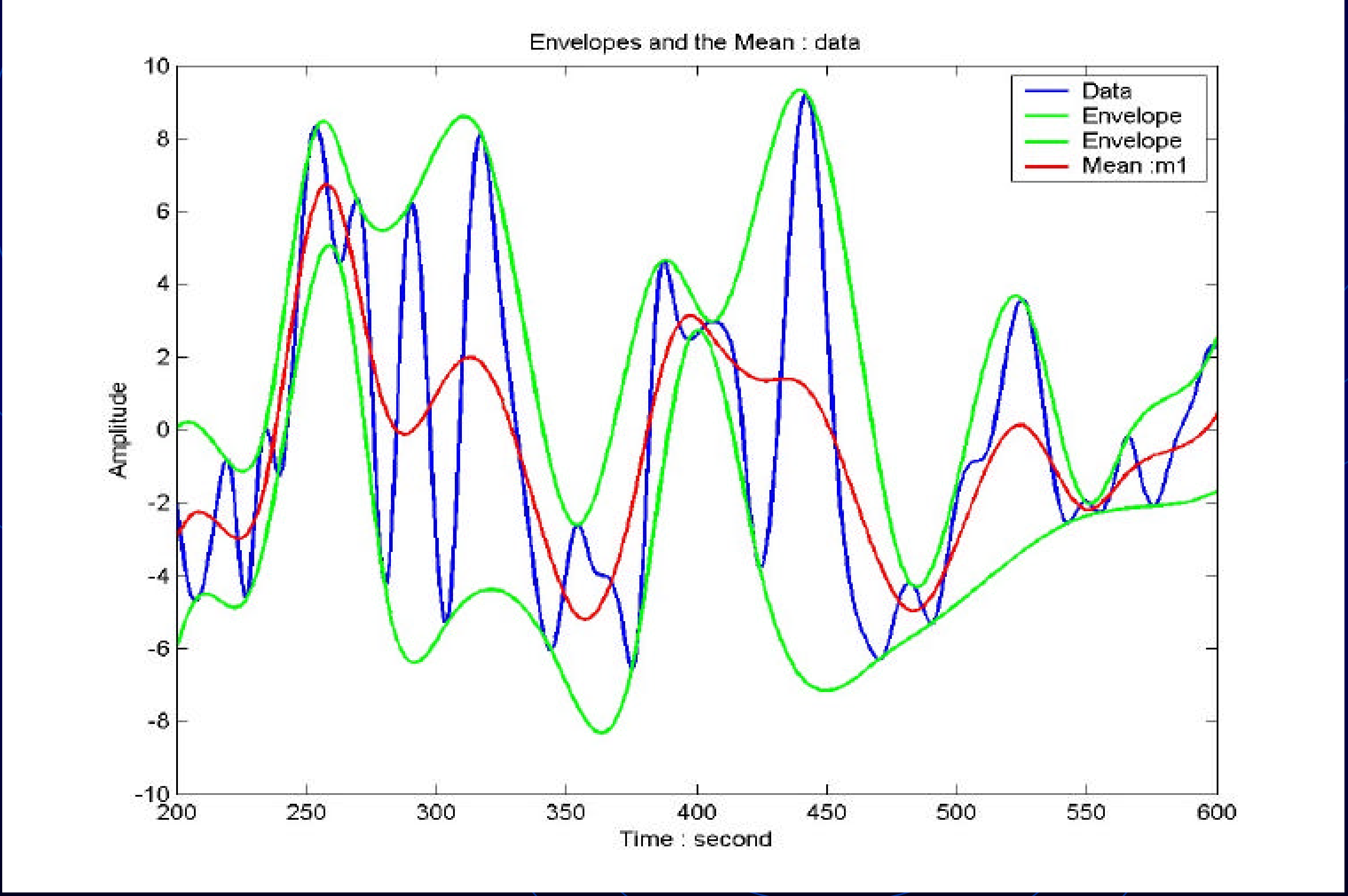
Sifting

Empirical Mode Decomposition:
Methodology : Test Data



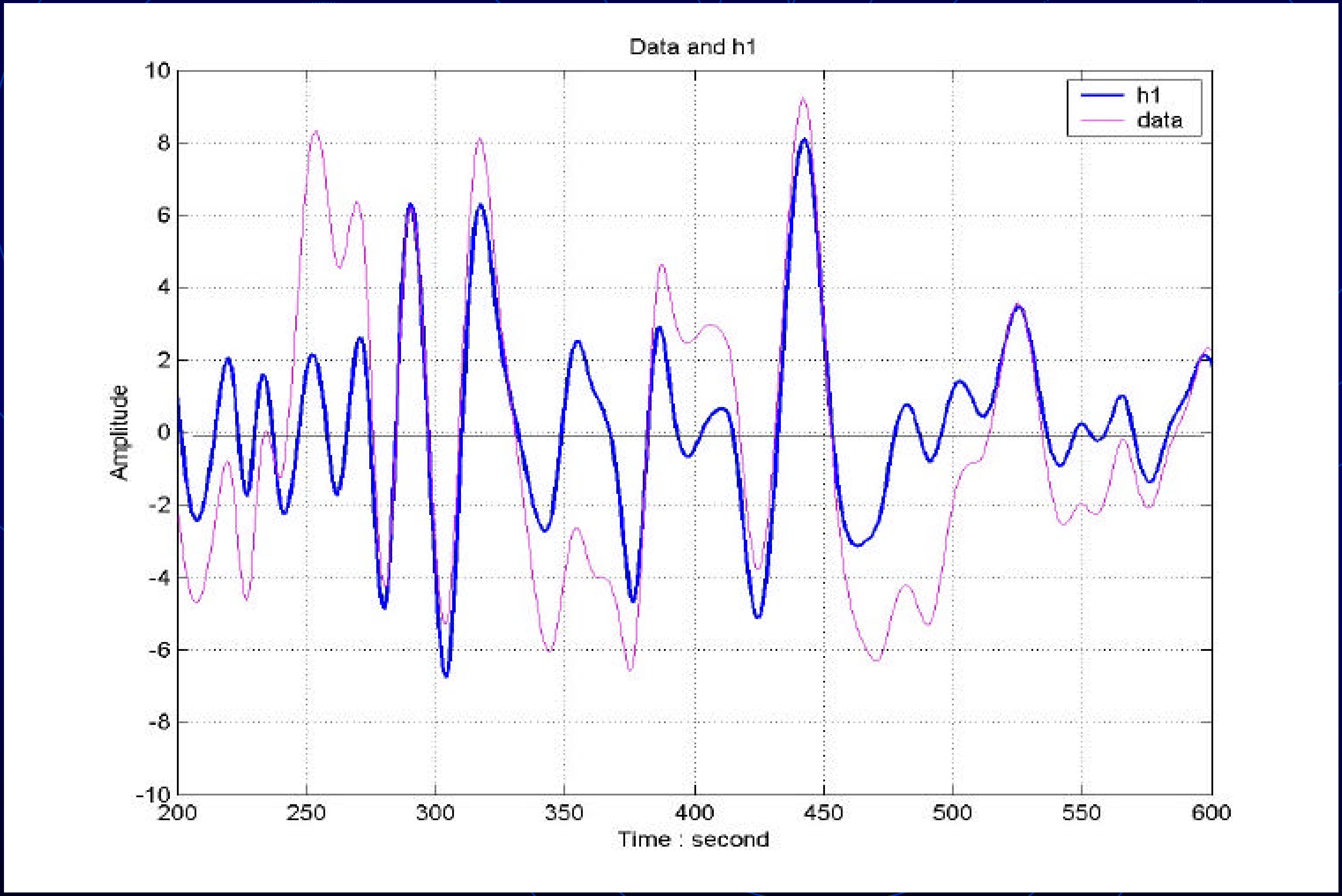
Empirical Mode Decomposition:

Methodology : data and m1

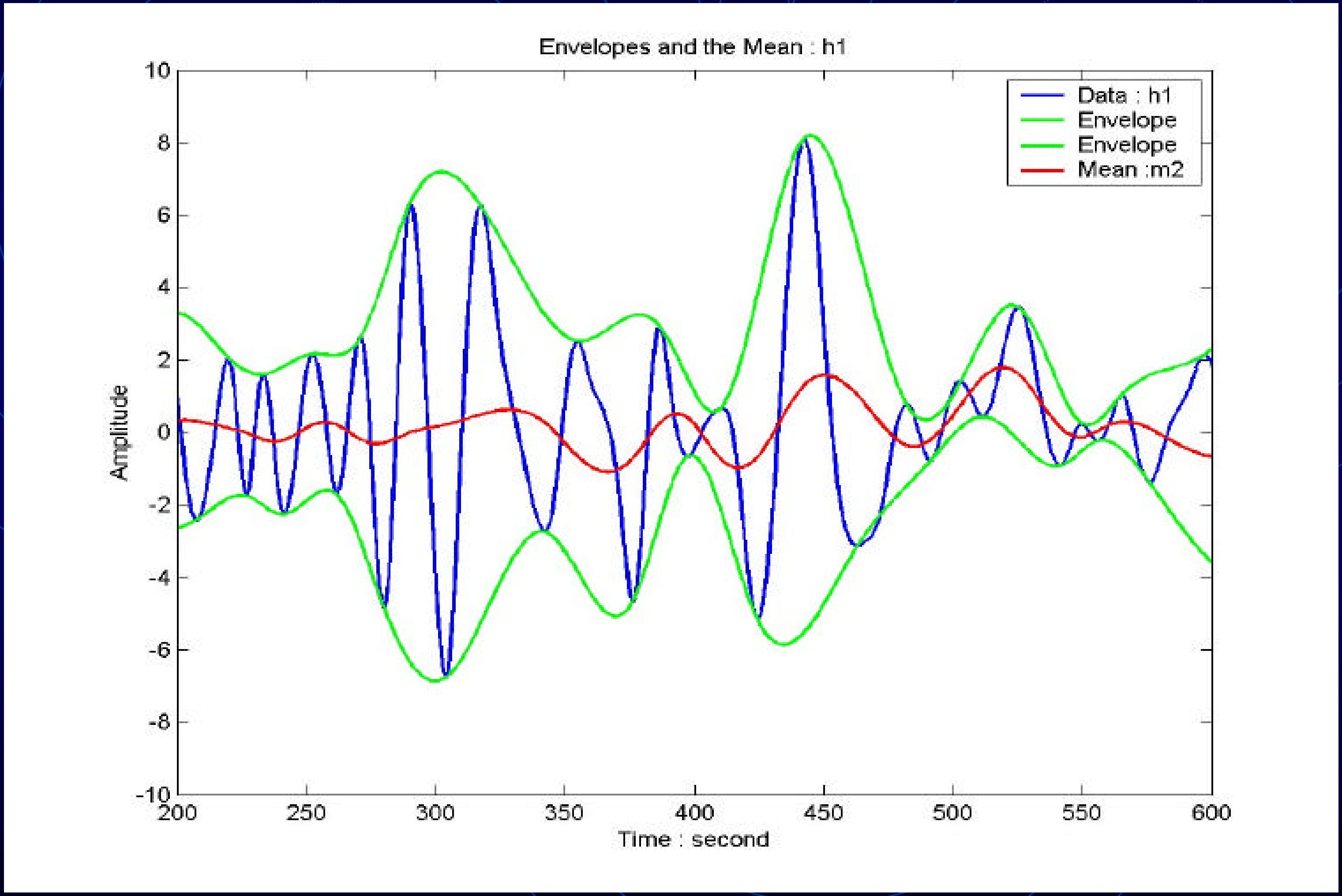


Empirical Mode Decomposition:

Methodology : data & h1

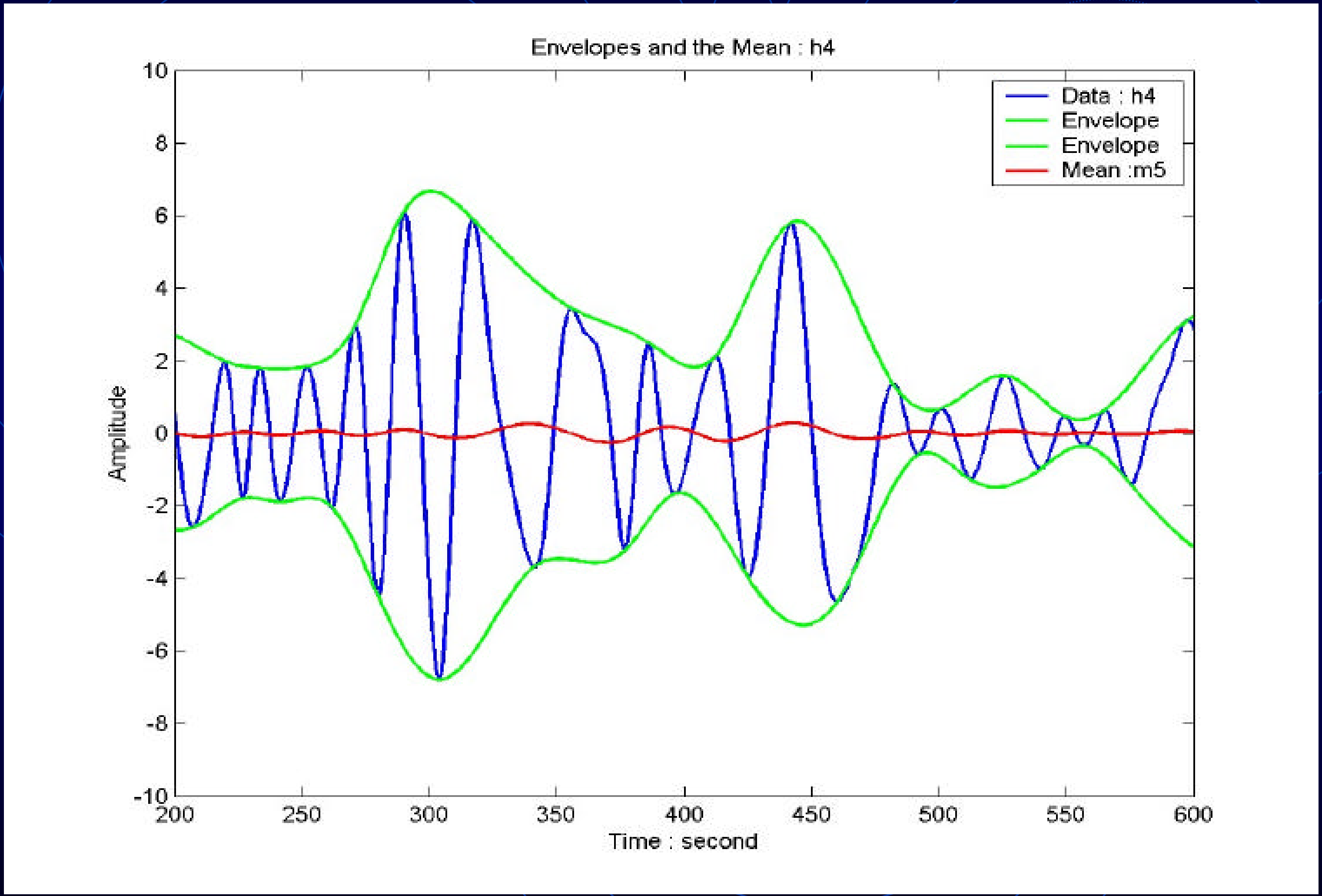


Empirical Mode Decomposition:
Methodology : h1 & m2



Empirical Mode Decomposition:

Methodology : h4 & m5



Empirical Mode Decomposition

Sifting : to get one IMF component

$$x(t) - m_1 = h_1 ,$$

$$h_1 - m_2 = h_2 ,$$

$$\cdots\cdots$$

$$\cdots\cdots$$

$$h_{k-1} - m_k = h_k .$$

$$\Rightarrow \quad h_k = c_1 .$$

Two Stoppage Criteria : S and SD

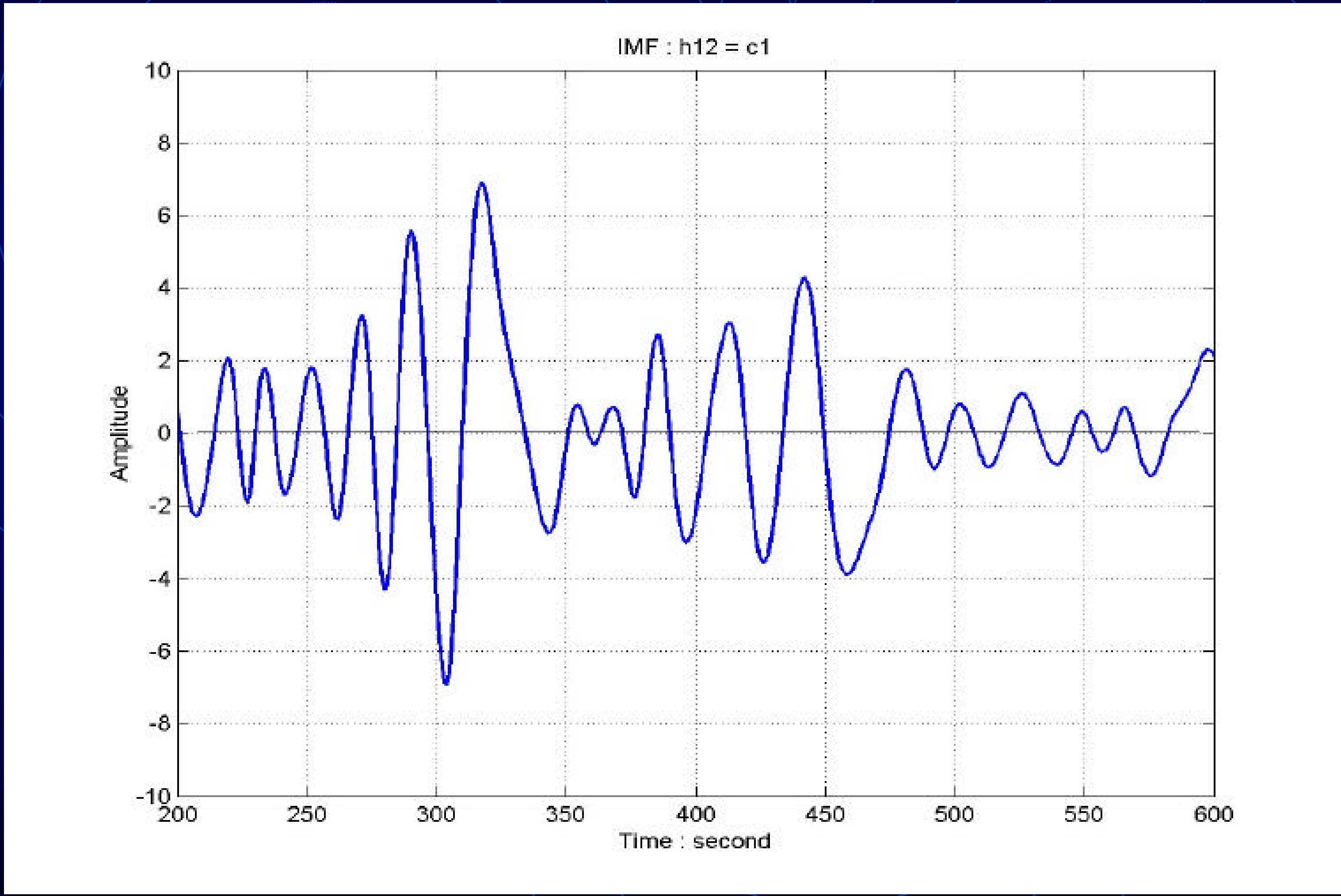
A. The S number : S is defined as the consecutive number of siftings, in which the numbers of zero-crossing and extrema are the same for these S siftings.

B. SD is small than a pre-set value, where

$$SD = \sum_{t=0}^{T} \frac{\left| h_{k-1}(t) - h_k(t) \right|^2}{h_{k-1}^{\,2}(t)} .$$

Empirical Mode Decomposition:

Methodology : IMF c1



Empirical Mode Decomposition

<u>Sifting : to get all the IMF components</u>

$$x(t) - c_1 = r_1 \ ,$$

$$r_1 - c_2 = r_2 \ ,$$

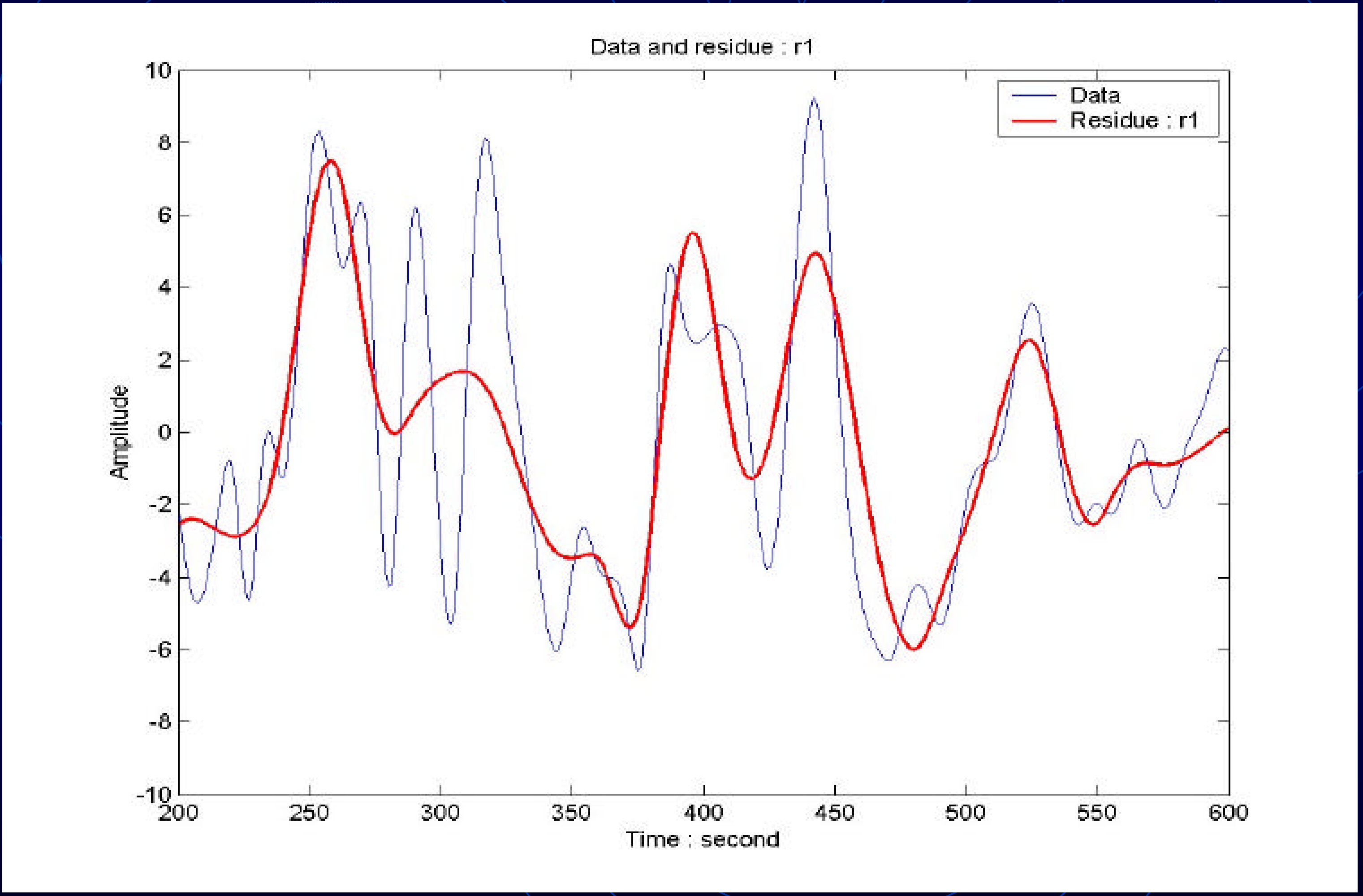
$$\cdots$$

$$r_{n-1} - c_n = r_n \ .$$

$$\Rightarrow \ x(t) - \sum_{j=1}^{n} c_j = r_n \ .$$

Empirical Mode Decomposition:

Methodology : data & r1



Hilbert Transform : Definition

For any $x(t) \in L^p$,

$$y(t) = \frac{1}{\pi} \wp \int_{\tau} \frac{x(\tau)}{t-\tau} \, d\tau \ ,$$

then, $x(t)$ *and* $y(t)$ *are complex conjugate :*

$$z(t) = x(t) + i\,y(t) = a(t)\, e^{i\theta(t)} \ ,$$

where

$$a(t) = \left(x^2 + y^2\right)^{1/2} \ \text{and} \ \theta(t) = \tan^{-1}\frac{y(t)}{x(t)} \ .$$

Comparison between FFT and HHT

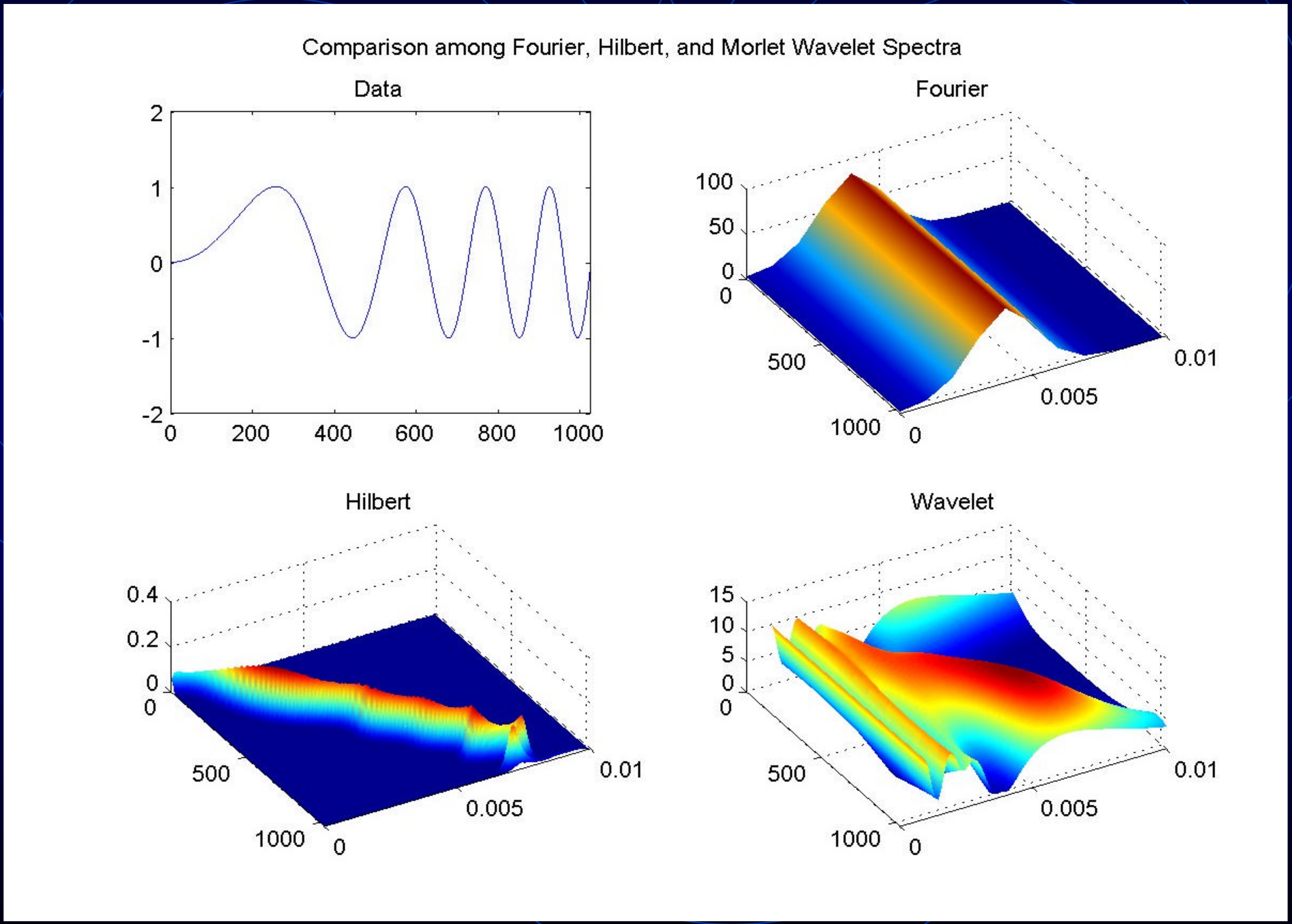
1. FFT :

$$x(t) = \Re \sum_j a_j e^{i w_j t} .$$

2. HHT :

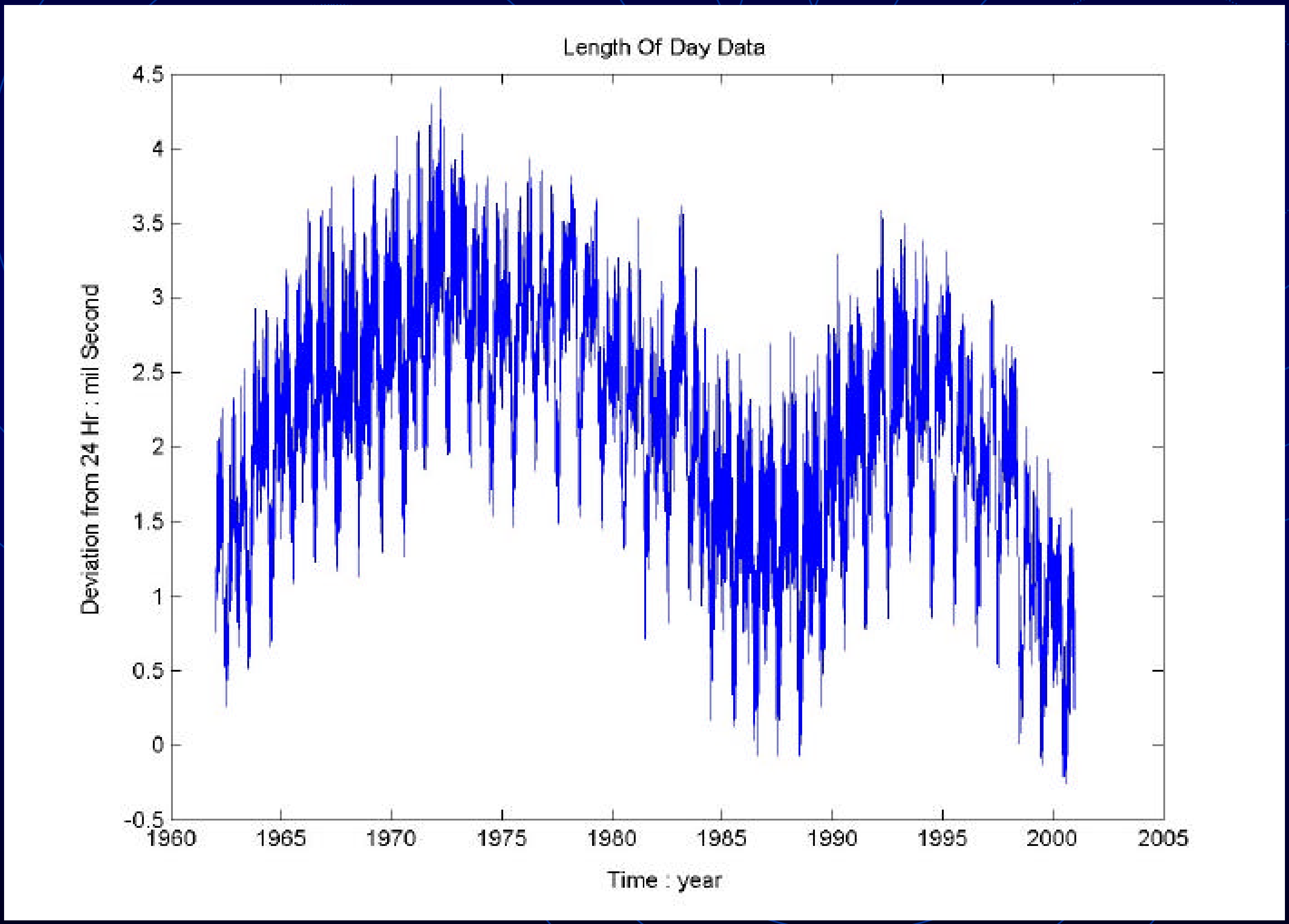
$$x(t) = \Re \sum_j a_j(t)\, e^{i \int_t w_j(t)\, dt} .$$

Comparisons: Fourier, Hilbert & Wavelet

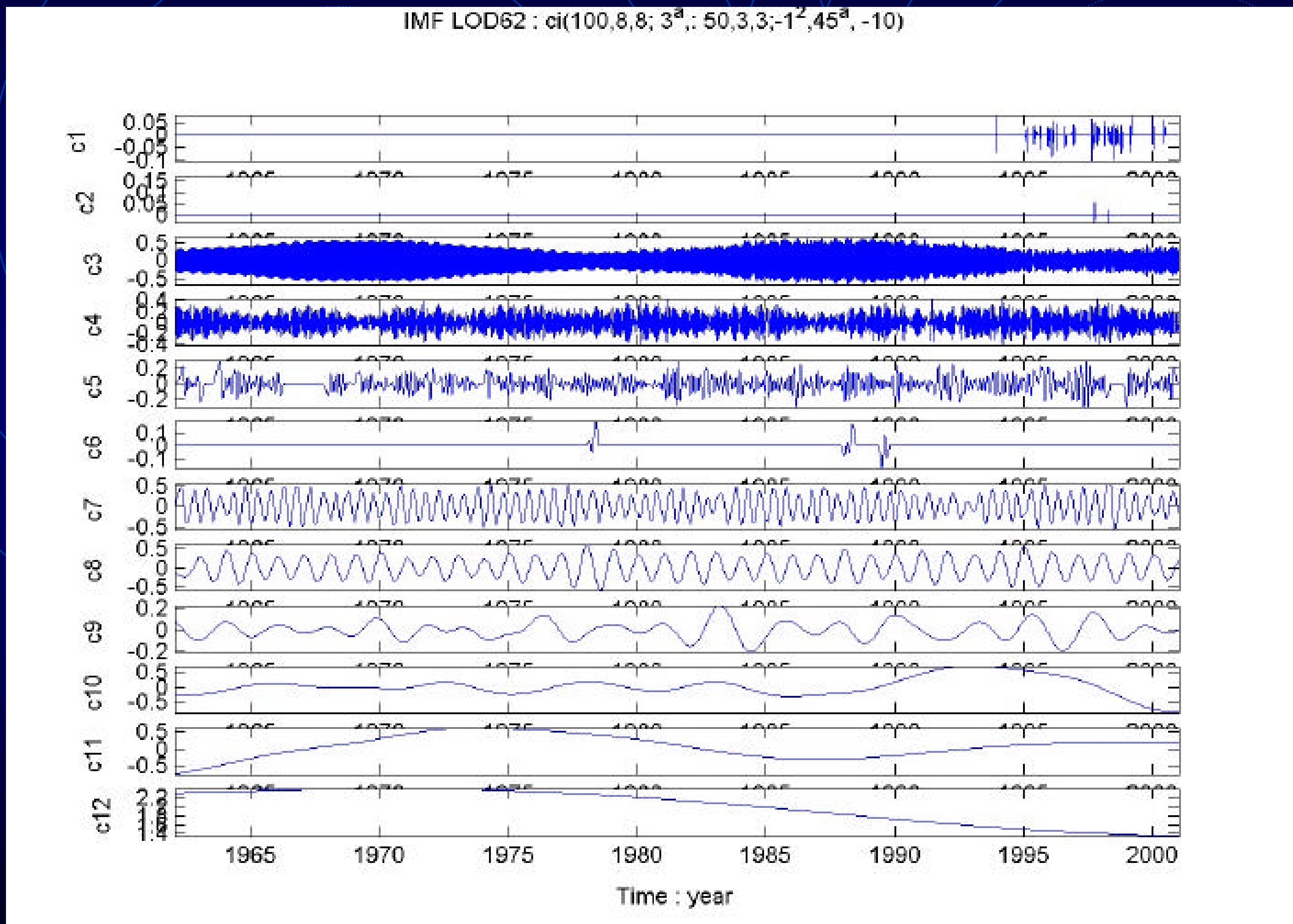


An Example of Sifting

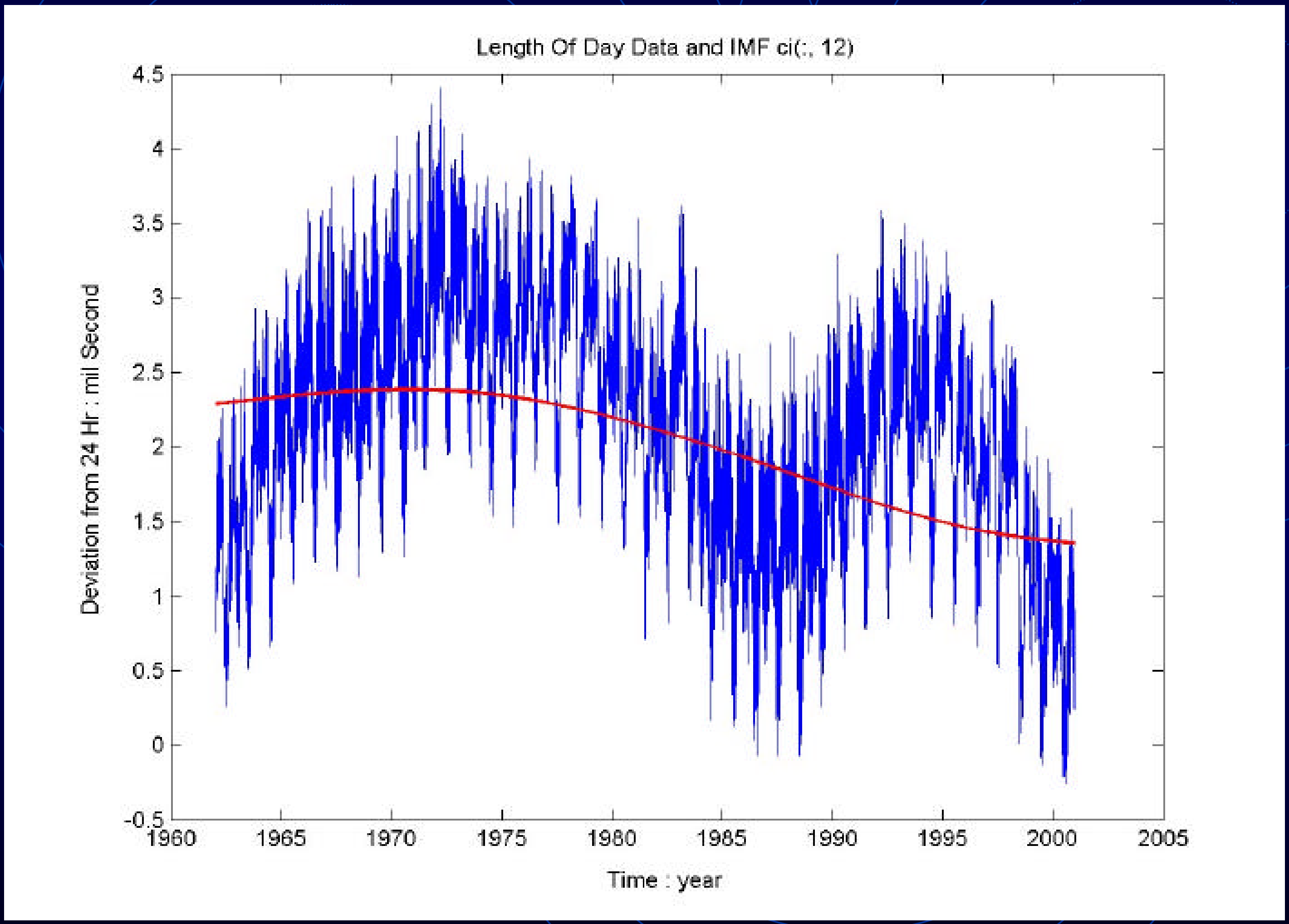
Length Of Day Data



LOD： IMF

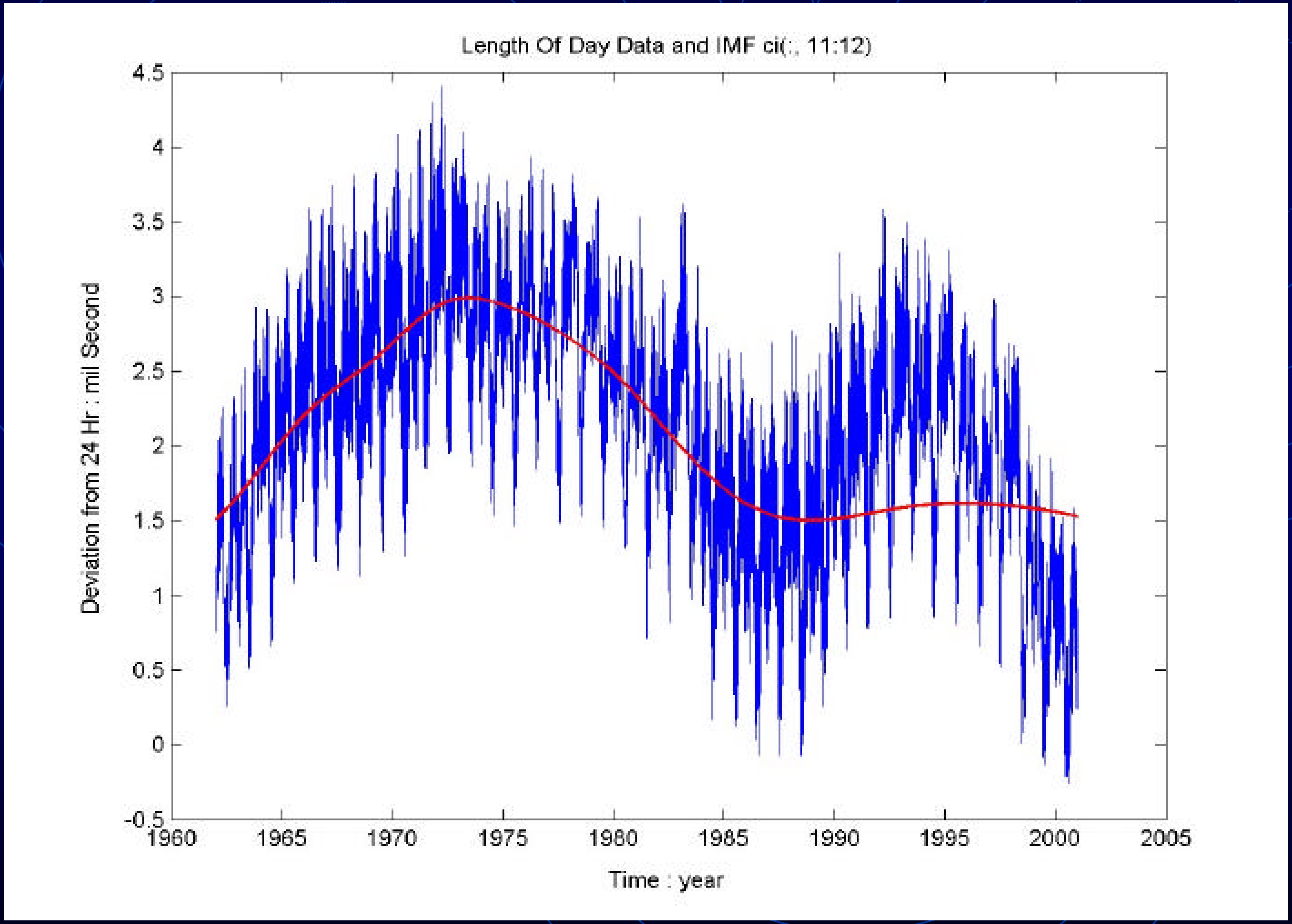

IMF LOD62 : ci(100,8,8; 3a,: 50,3,3;-12,45a, -10)
c1
c2
c3
c4
c5
c6
c7
c8
c9
c10
c11
c12
1965
1970
1975
1980
1985
1990
1995
2000
Time : year

LOD : Data & c12

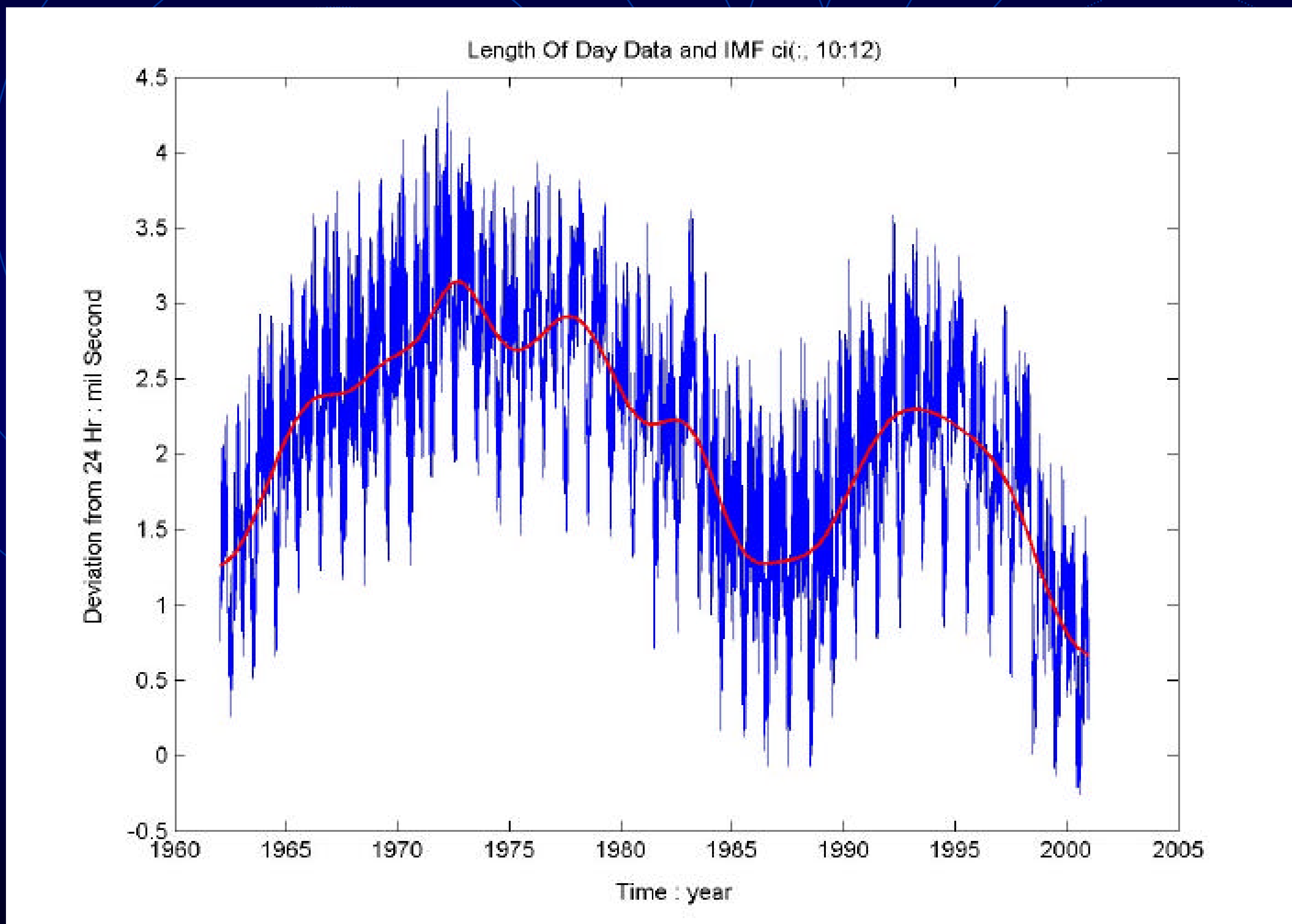


Length Of Day Data and IMF ci(:, 12)
Deviation from 24 Hr : mil Second
Time : year
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

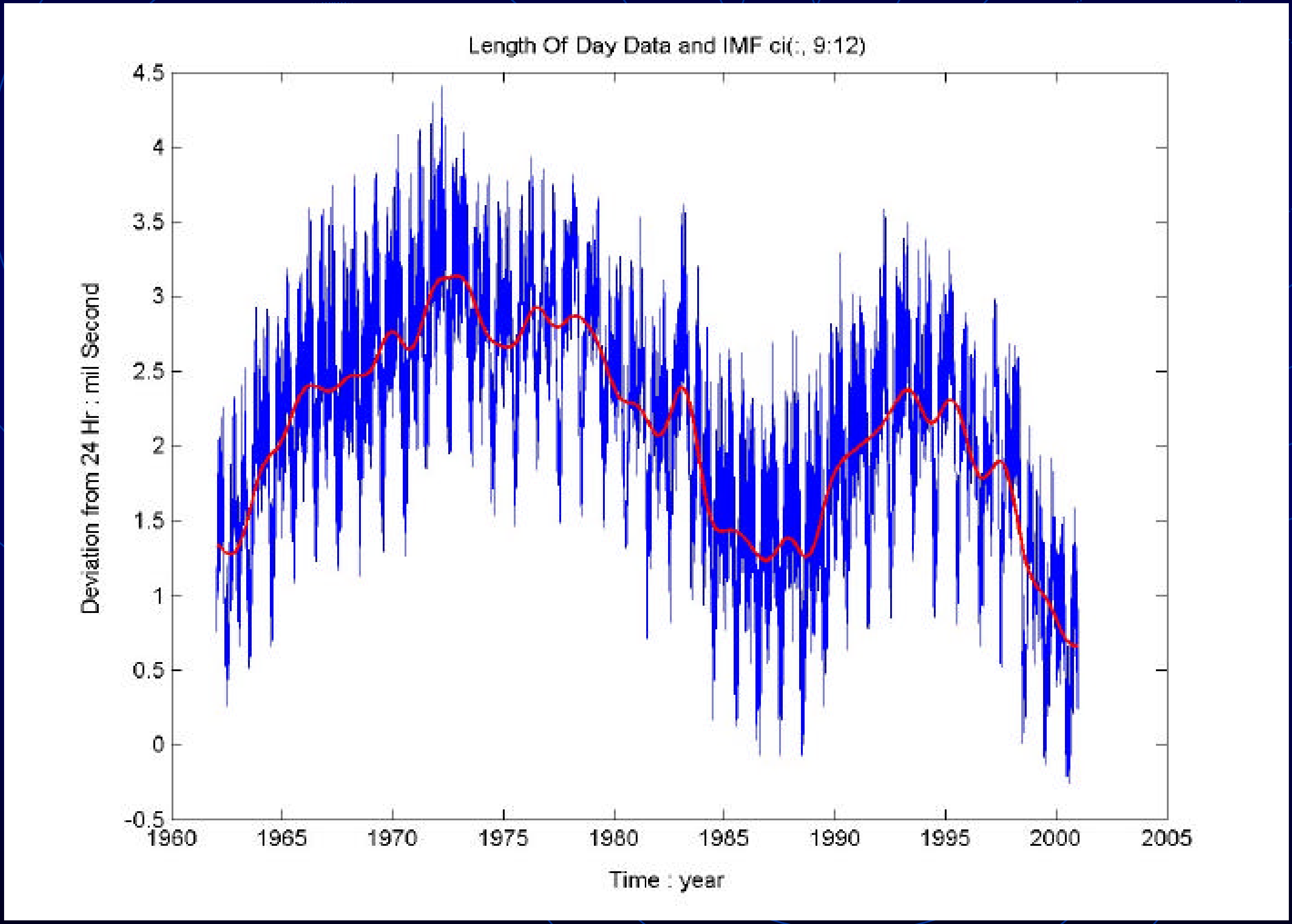
LOD：Data & Sum c11-12



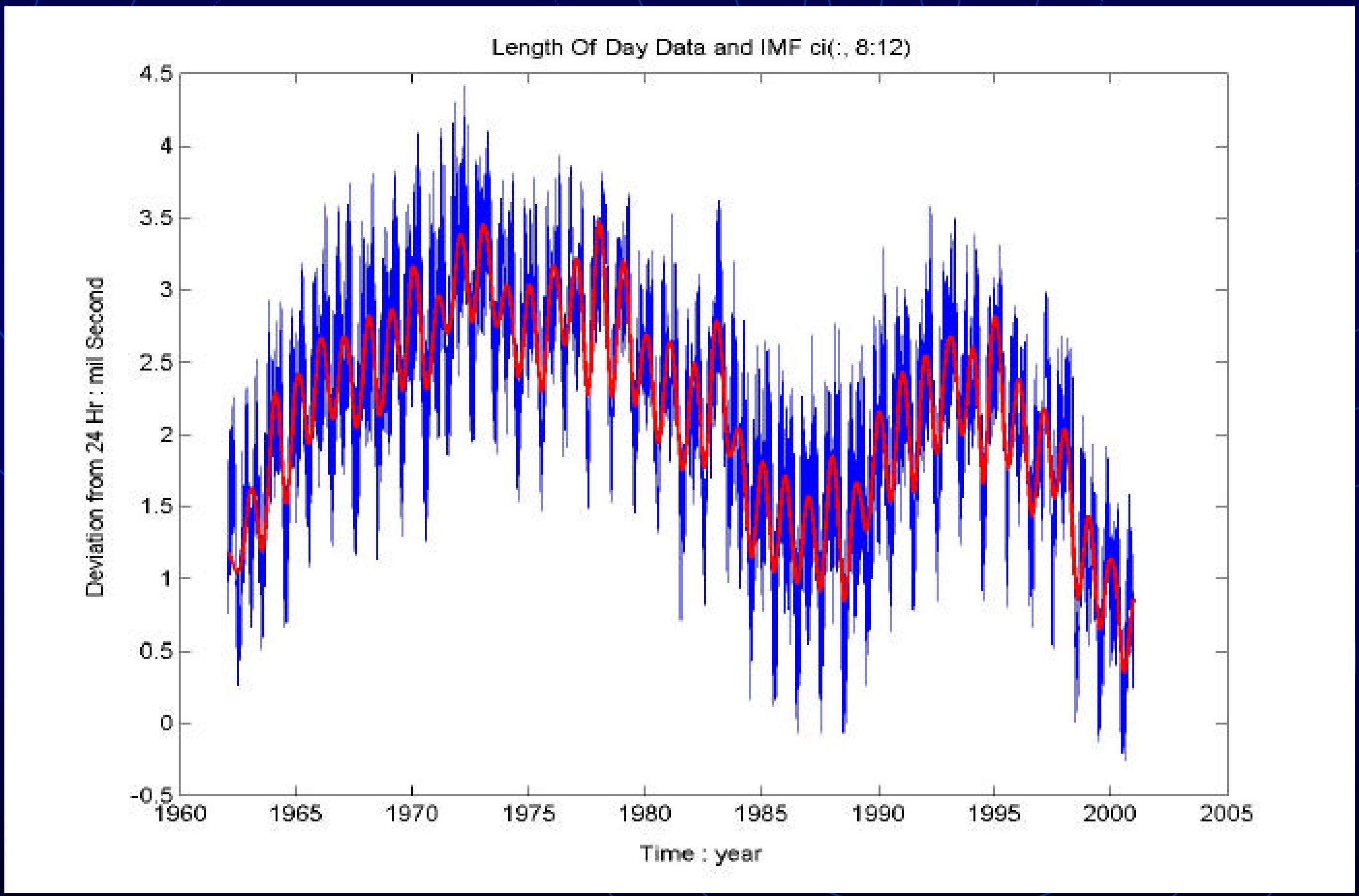
LOD : Data & sum c10-12



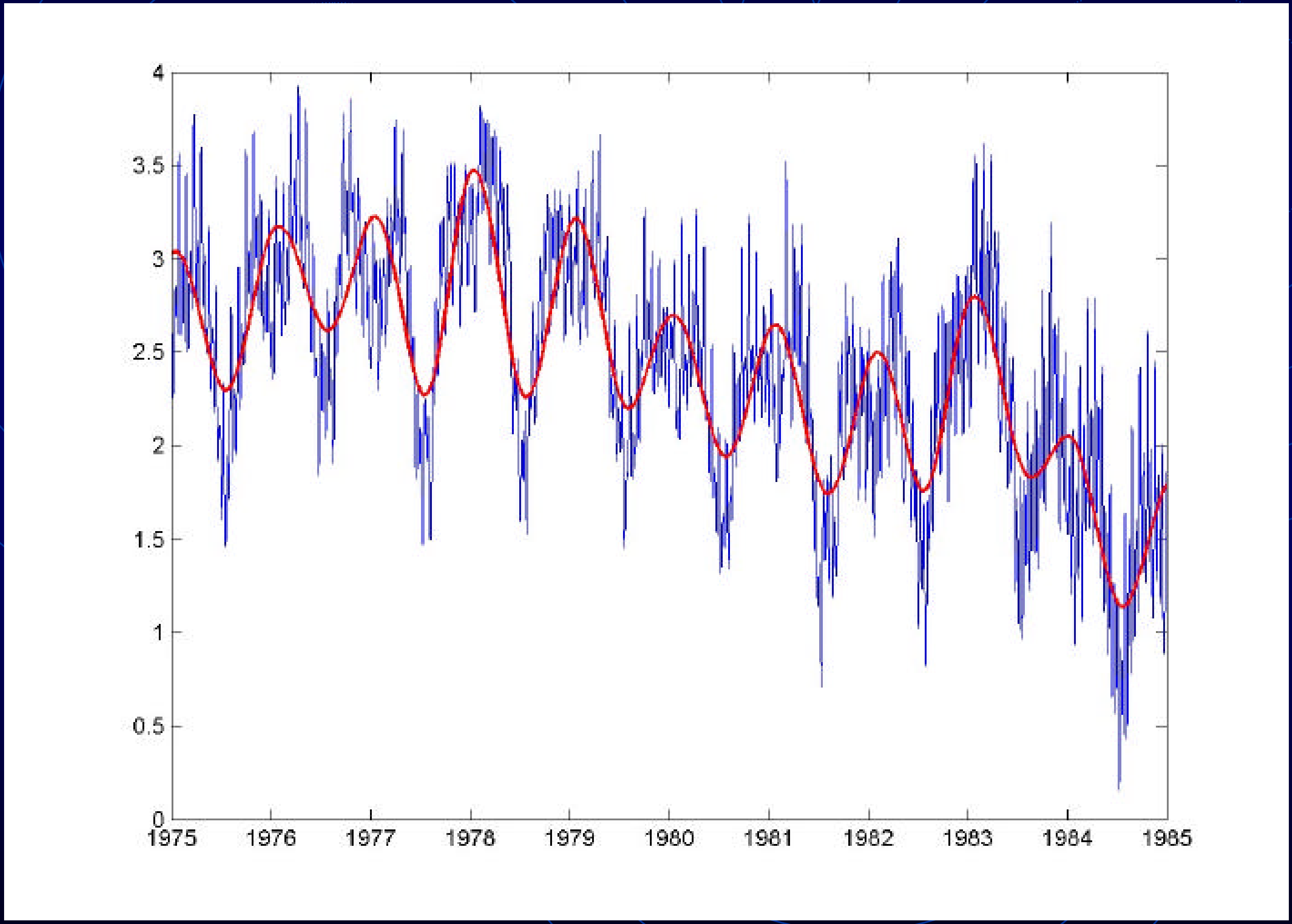
LOD : Data & c9 - 12



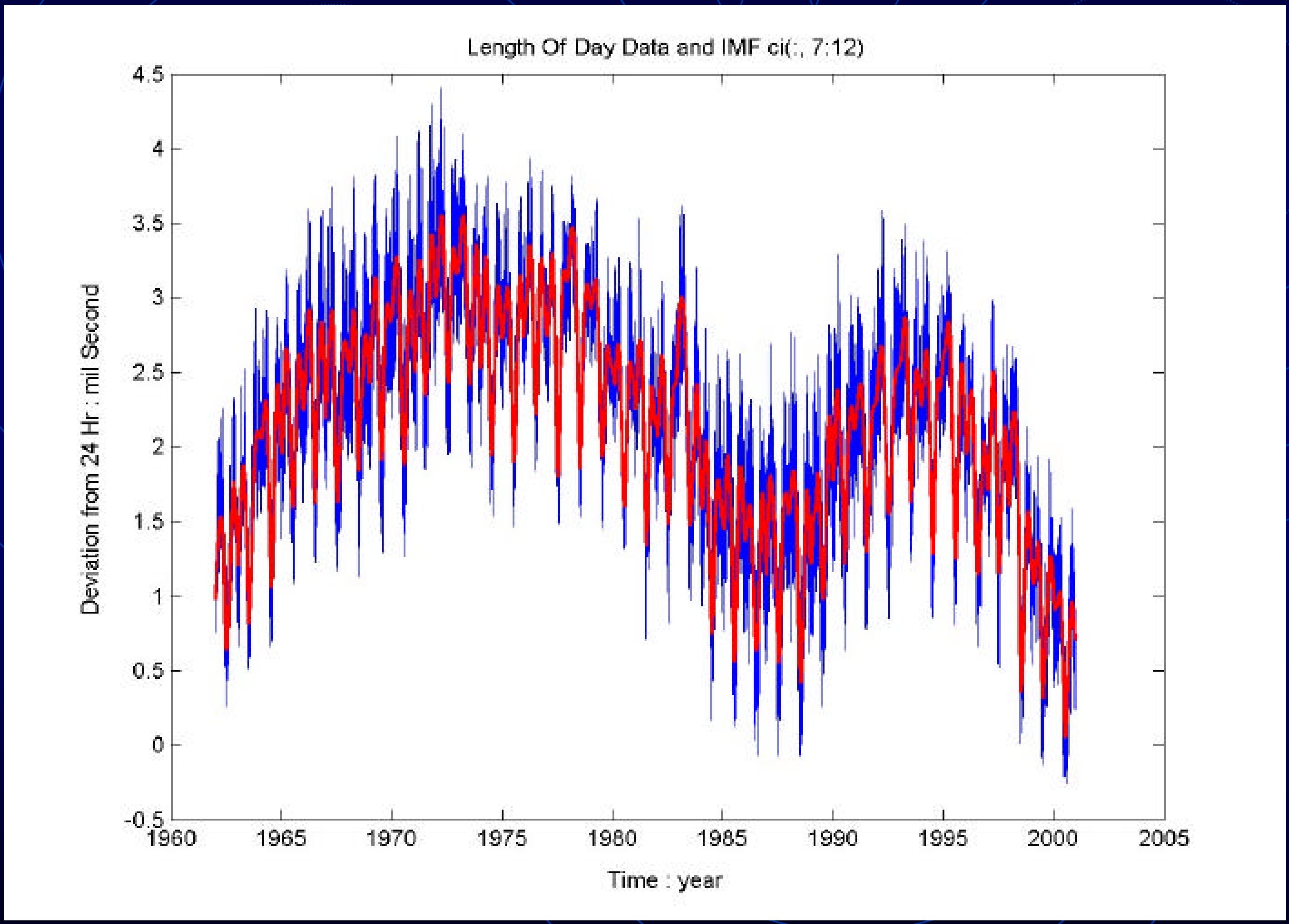
LOD : Data & c8 - 12



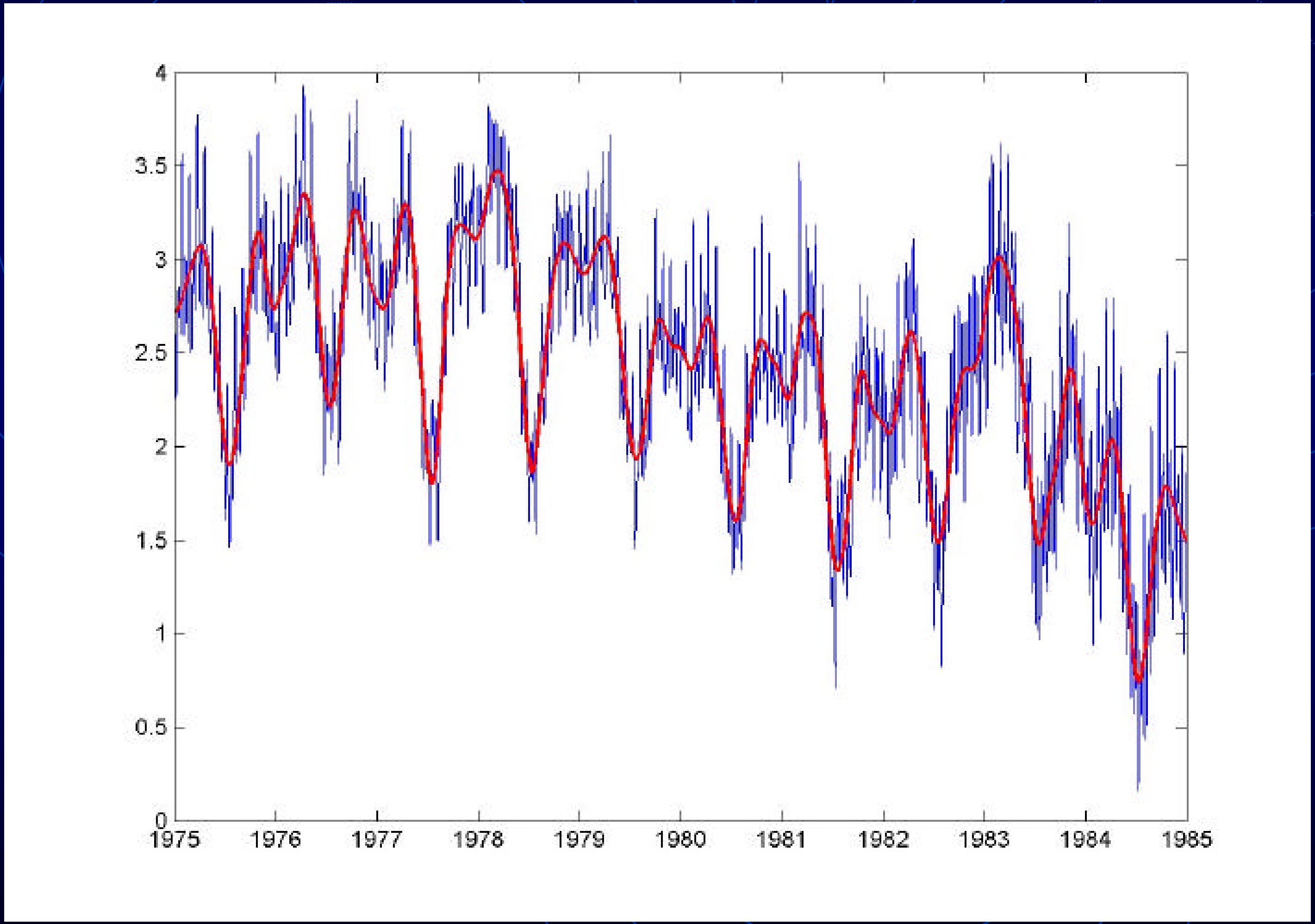
LOD：Detailed Data and Sum c8-c12



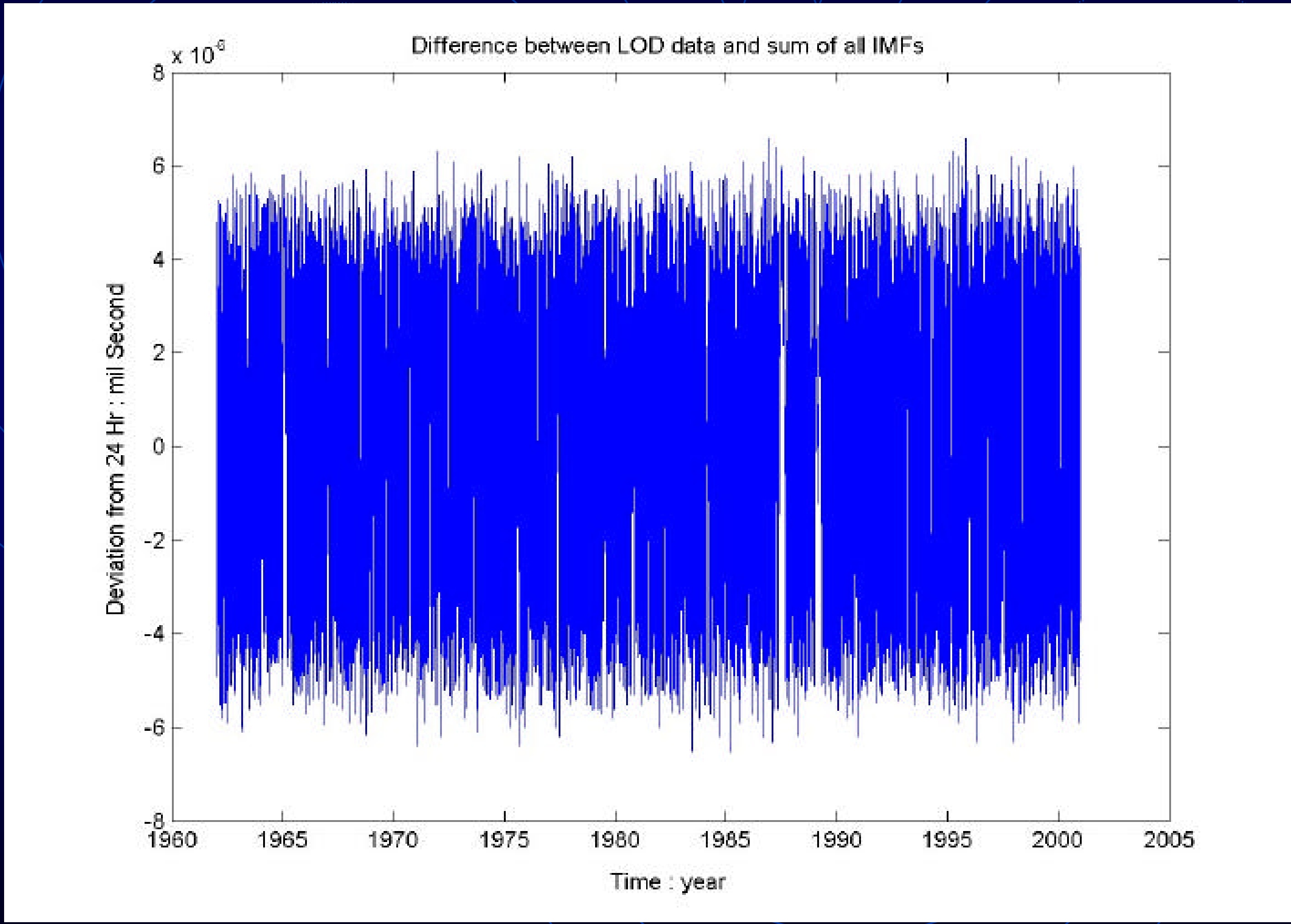
LOD : Data & c7 - 12



LOD：Detail Data and Sum IMF c7-c12

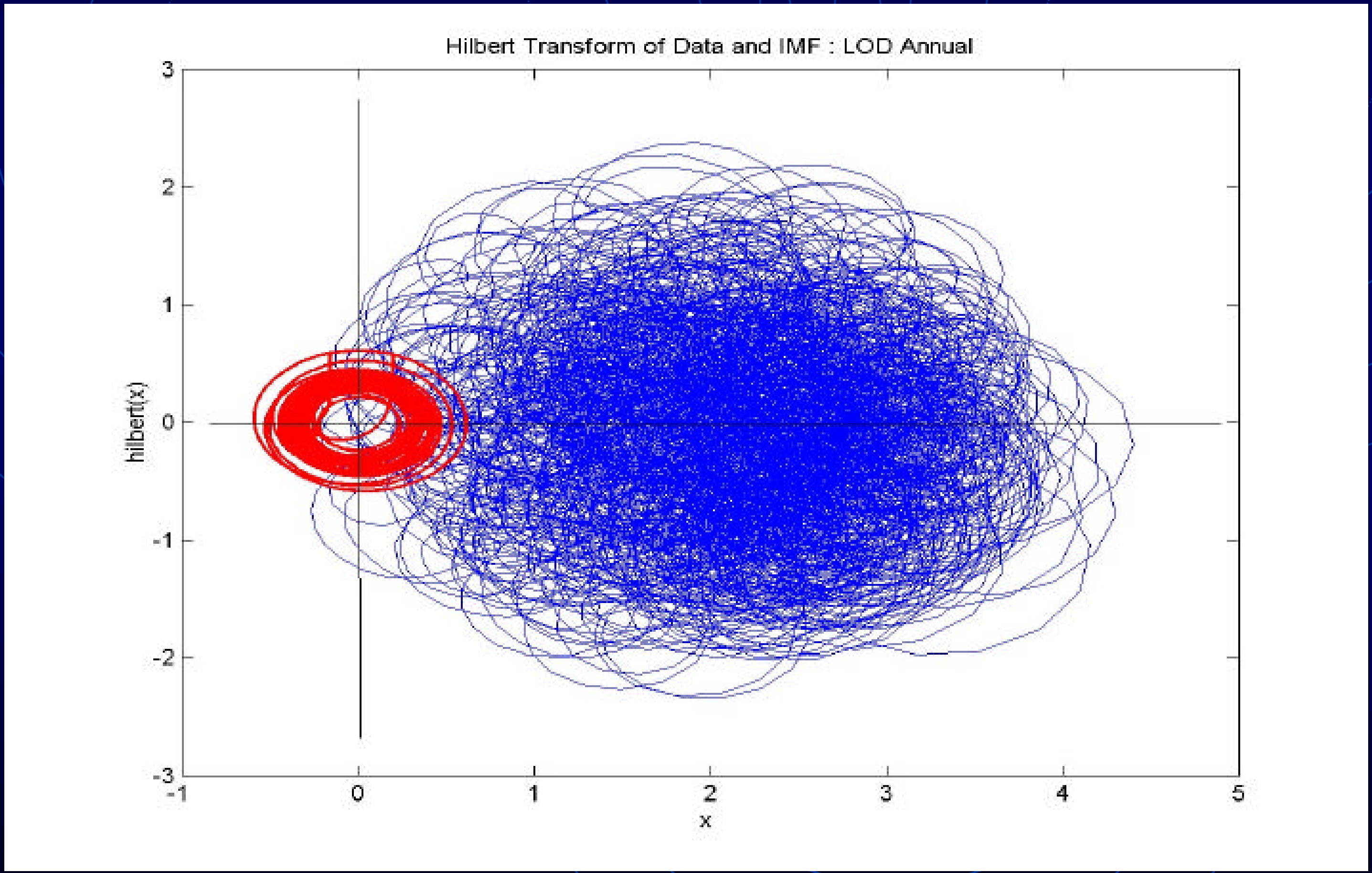


LOD：Difference Data – sum all IMFs

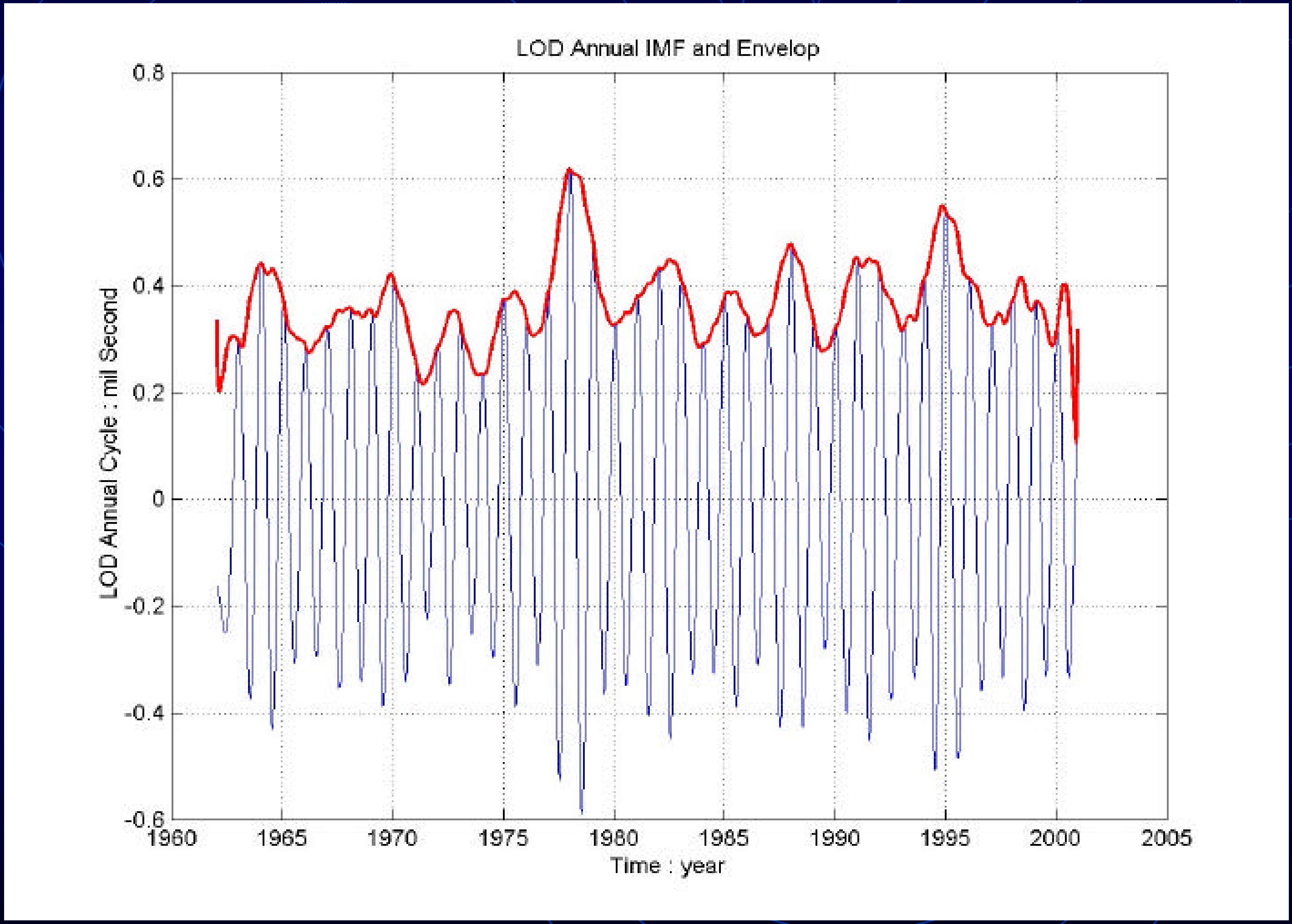


Traditional View

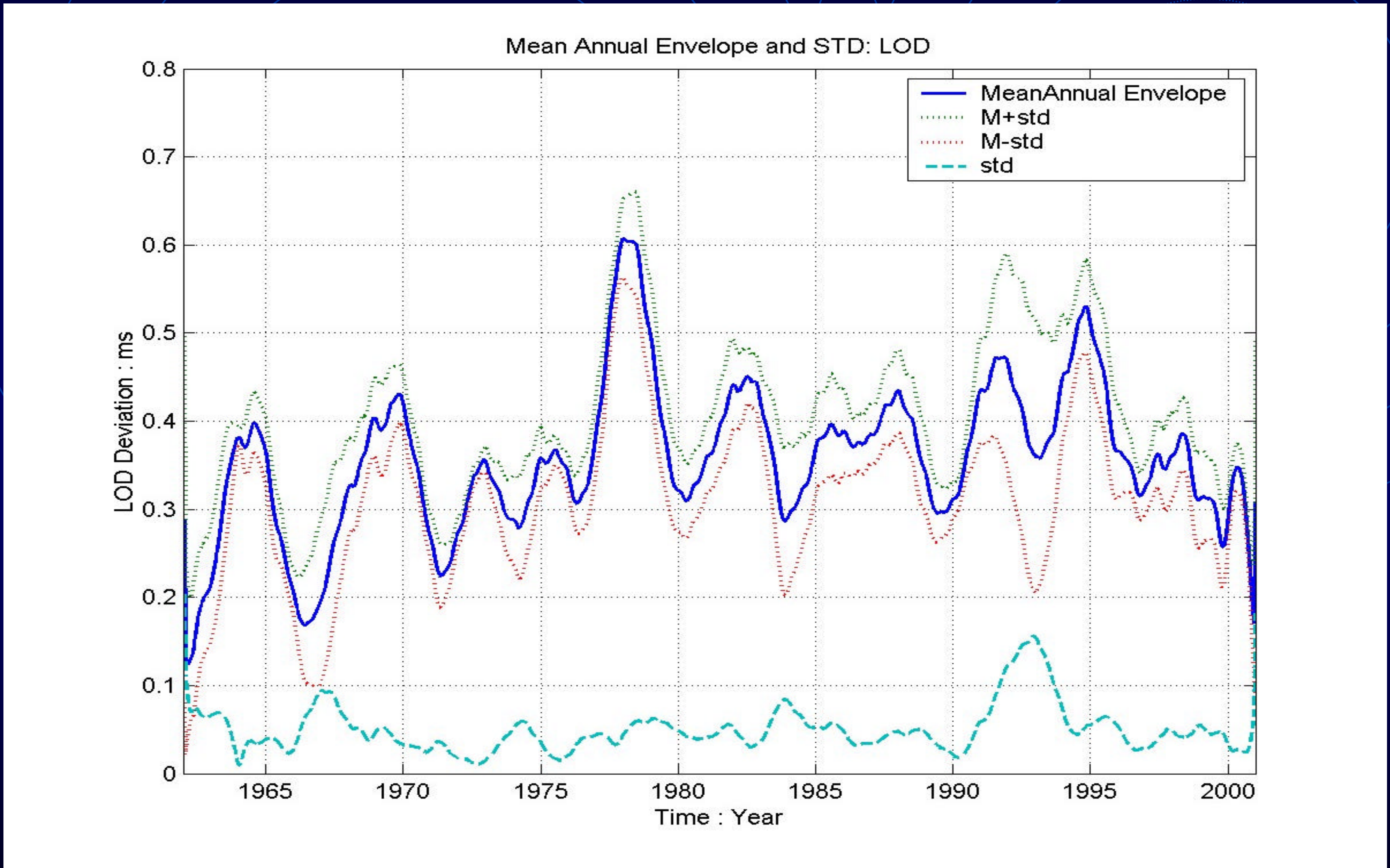
a la Hahn (1995) : Hilbert



LOD : Mean envelop from 11 different siftings



Mean Envelopes for Annual Cycle IMFs



Comparisons

	Fourier	Wavelet	Hilbert
Basis	a priori	a priori	Adaptive
Frequency	Convolution: Global	Convolution: Regional	Differentiation: Local
Presentation	Energy-frequency	Energy-time-frequency	Energy-time-frequency
Nonlinear	no	no	yes
Non-stationary	no	yes	yes
Feature extraction	no	discrete : no continuous: yes	yes

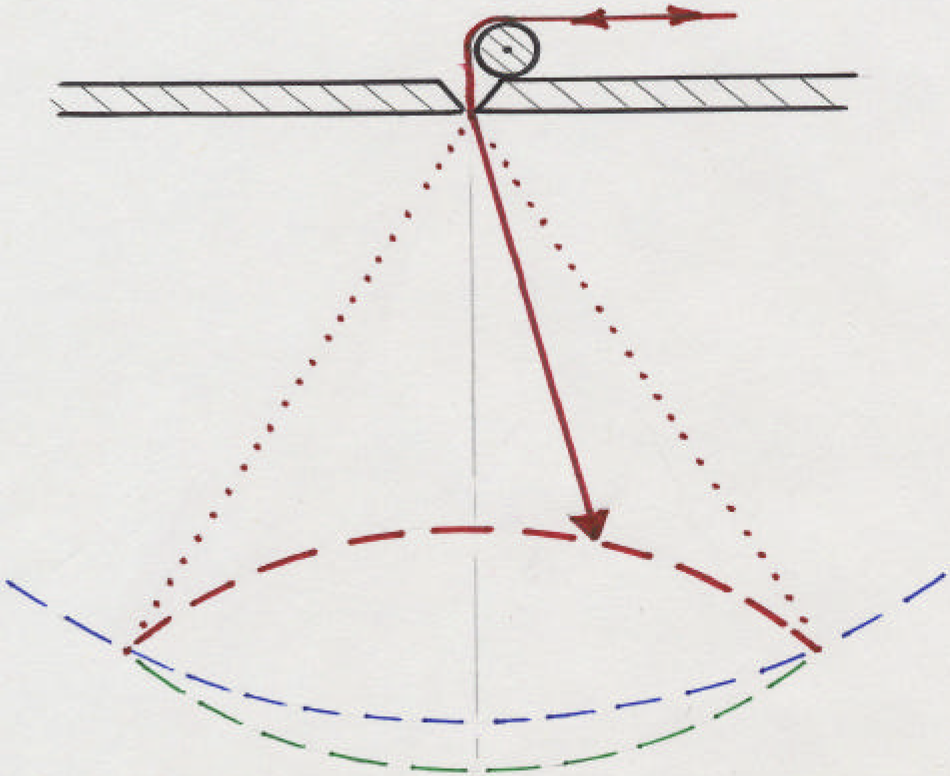
Characteristics of Data from Nonlinear Processes

$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = \gamma \cos \omega t$$

$$\Rightarrow \quad \frac{d^2 x}{dt^2} + x \left(1 + \varepsilon x^2\right) = \gamma \cos \omega t$$

$\Rightarrow$ *Spring with position dependent constant,*
intra-wave frequency modulation;
therefore, we need instantaneous frequency.

Duffing Pendulum



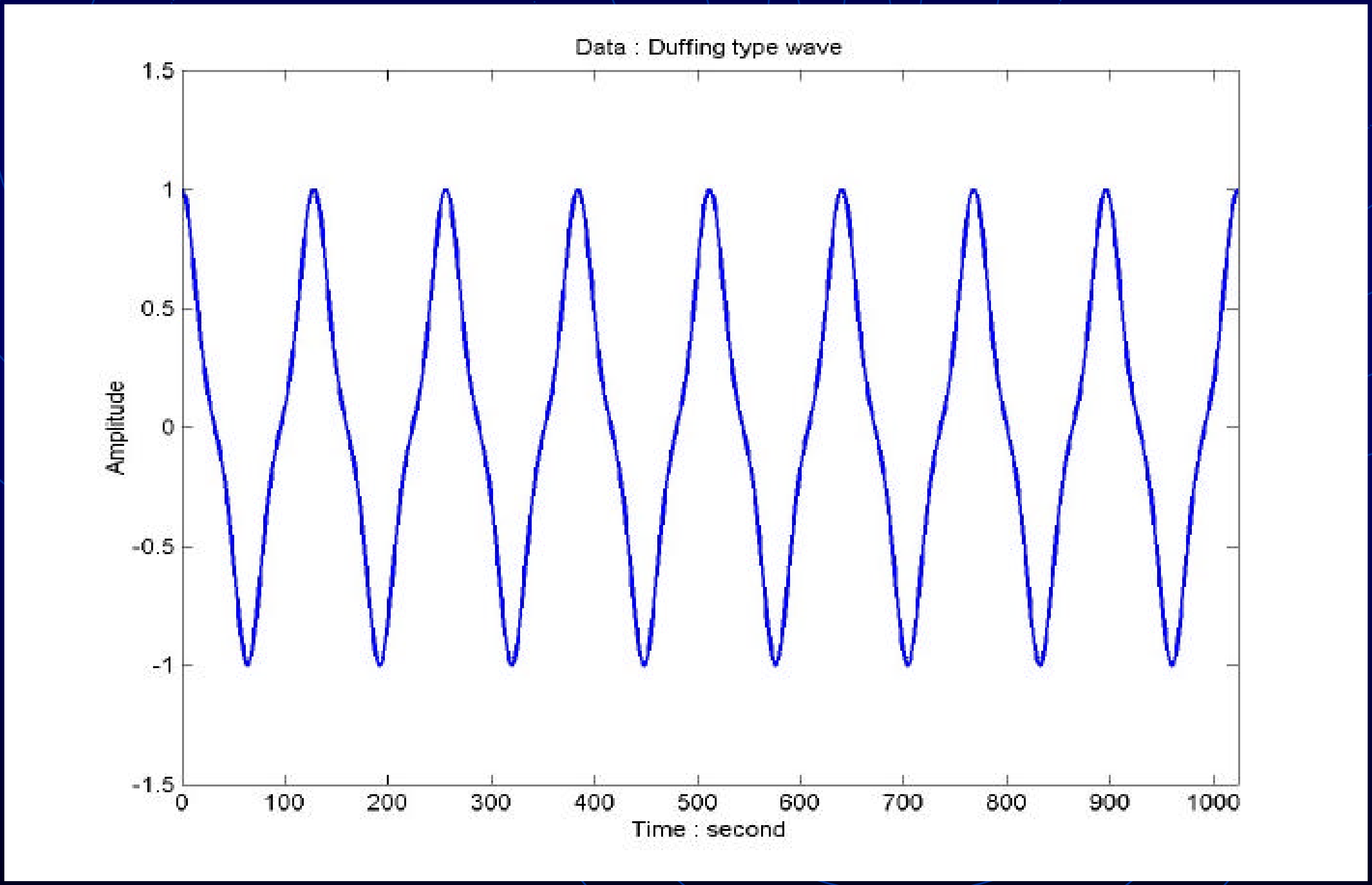
$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = a \cos \gamma t$$

$$\frac{d^2 x}{dt^2} + x (1 + \varepsilon x^2) = a \cos \gamma t$$

Hilbert’s View on Nonlinear Data

Duffing Type Wave

Data: $x = cos(wt + 0.3\ sin2wt)$



Duffing Type Wave

Perturbation Expansion

For $e = 1$, *we can have*

$$x(t) = \cos\left(wt + e\sin 2wt\right)$$

$$= \cos wt \cos\left(e\sin 2wt\right) - \sin wt \sin\left(e\sin 2wt\right)$$

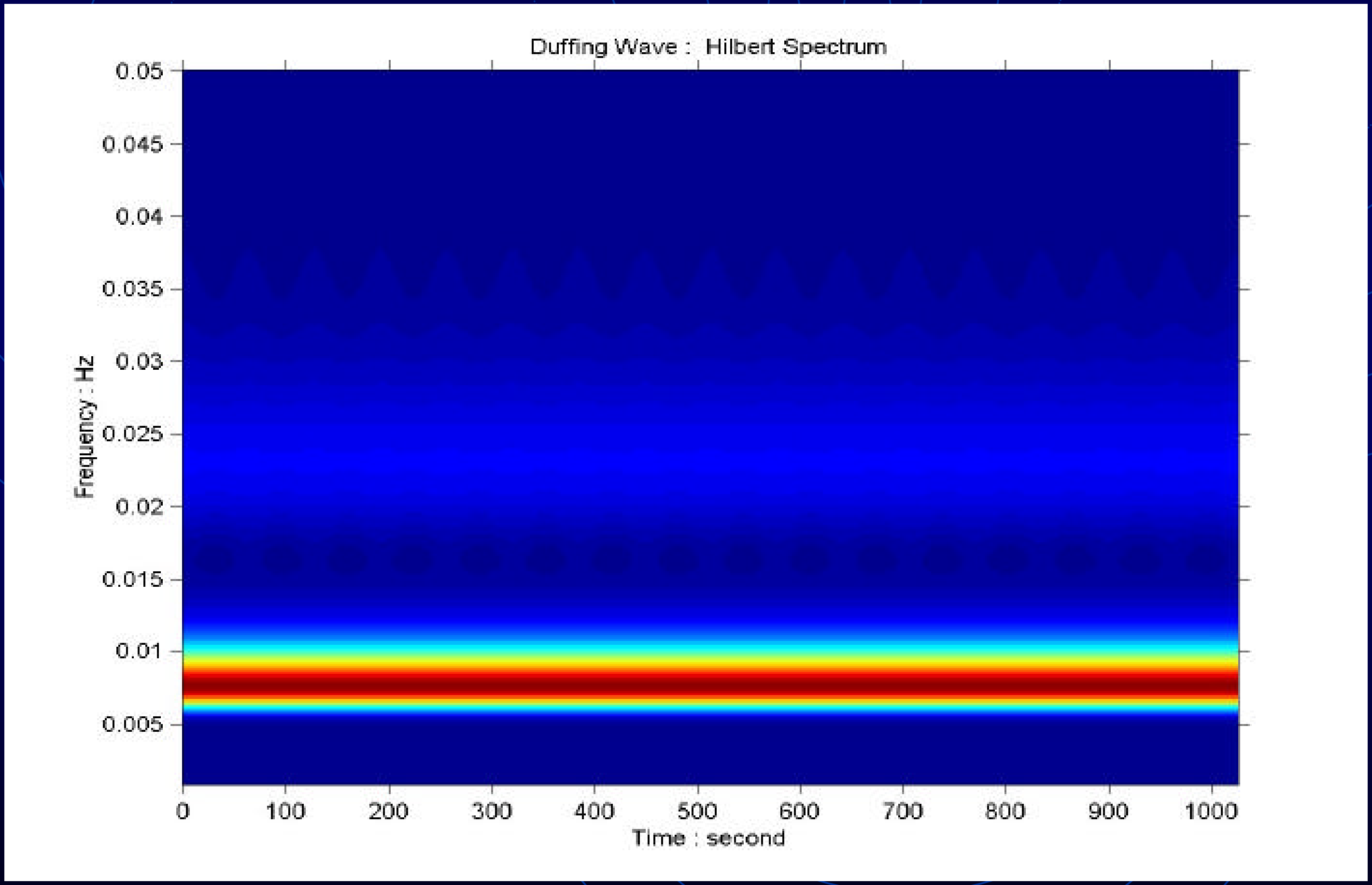
$$= \cos wt - e\sin wt \sin 2wt + \ldots.$$

$$= \left(1 - \frac{e}{2}\right)\cos wt + \frac{e}{2}\cos 3wt + \ldots.$$

This is very similar to the solution of Duffing equation .

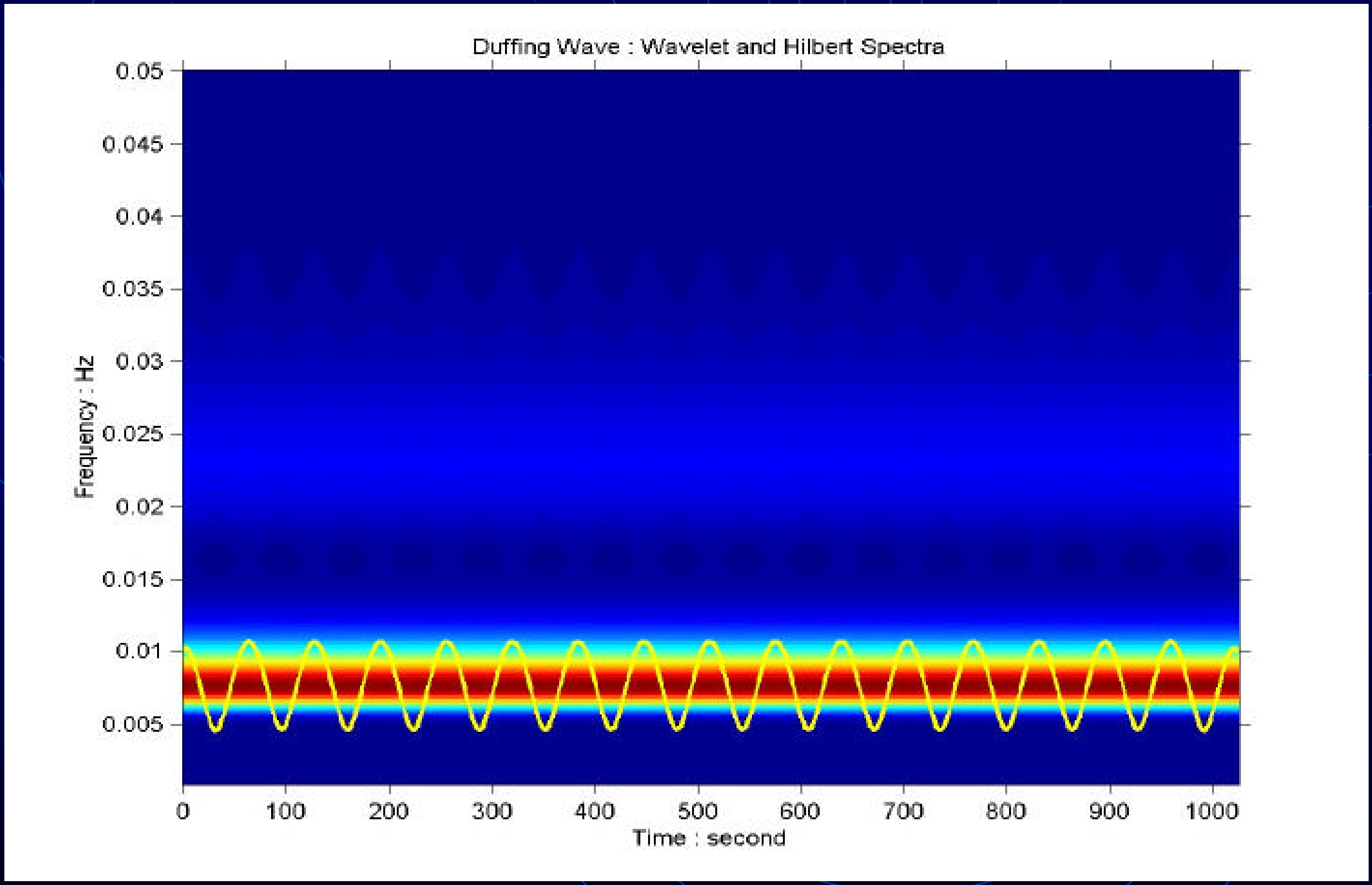
Duffing Type Wave

Wavelet Spectrum



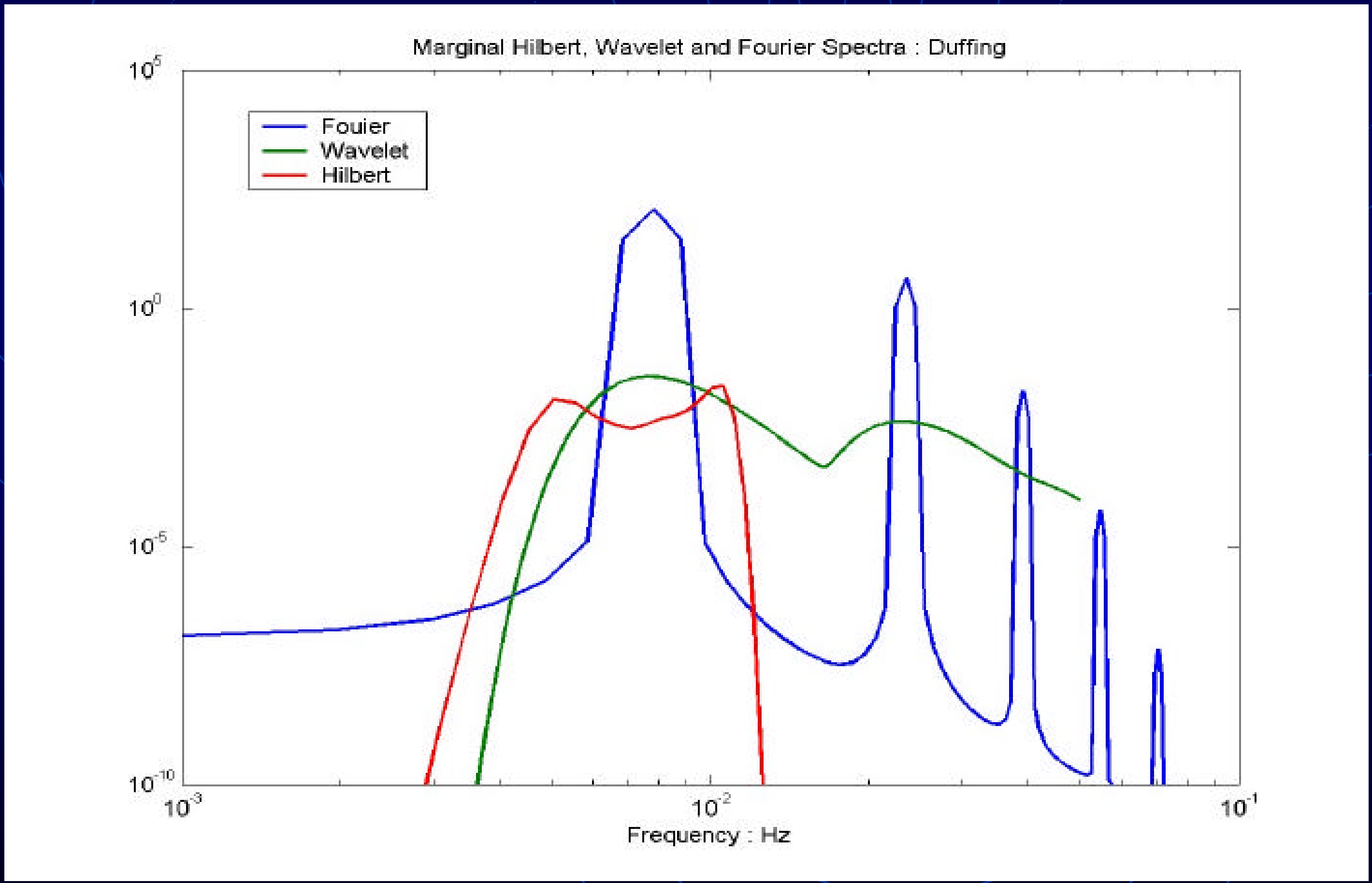
Duffing Type Wave

Hilbert Spectrum



Duffing Type Wave

Marginal Spectra



Technology Description

Results:

- **An adaptive basis to filter signal**
- **Frequency defined as a function of time by differentiation rather than convolution analysis**
- **Sharp identification of embedded structures**
- **A more simple and revealing interpretation than prior methods**

Market Potential

Key Considerations

- Conceptually simple and direct
- An efficient, adaptive, user-friendly set of algorithms
- Capable of analyzing nonlinear and nonstationary signals
- Improves accuracy by using an adaptive basis to preserve intrinsic properties of data
- Yields results with more physical meaning and a different perspective than existing tools
- Useful in analyzing a variety of from nonlinear and nonstationary processes

Possible Applications

- Vibration, speech and acoustic signal analyses : this also applies to machine health monitoring.
- Non-destructive test and structural Health monitoring
- Earthquake Engineering
- As a nonlinear Filter
- Bio-medical applications
- Time-Frequency-Energy distribution for general nonlinear and nonstationary data analysis, for example, turbulence

Sound Enhancement :

- Fourier filter is linear and stationary; it works in Frequency domain
- Fourier filter will take away harmonics and dull the sharp corners of all the fundamentals
- EMD filter is nonlinear and intermittent; it works in Time domain
- EMD filter will take the unwanted noise of short periods and leaves the fundamentals unchanged

EMD as Filter

$$\text{Once we have the EMD expansion:} \quad x(t) = \sum_{j=1}^{N} c_j \;,$$

we can define the filters as fellows:

$$\text{Low Pass Filter:} \quad x_L(t) = \sum_{j=L}^{N} c_j \;;$$

$$\text{High Pass Filter:} \quad x_H(t) = \sum_{j=1}^{H} c_j \;;$$

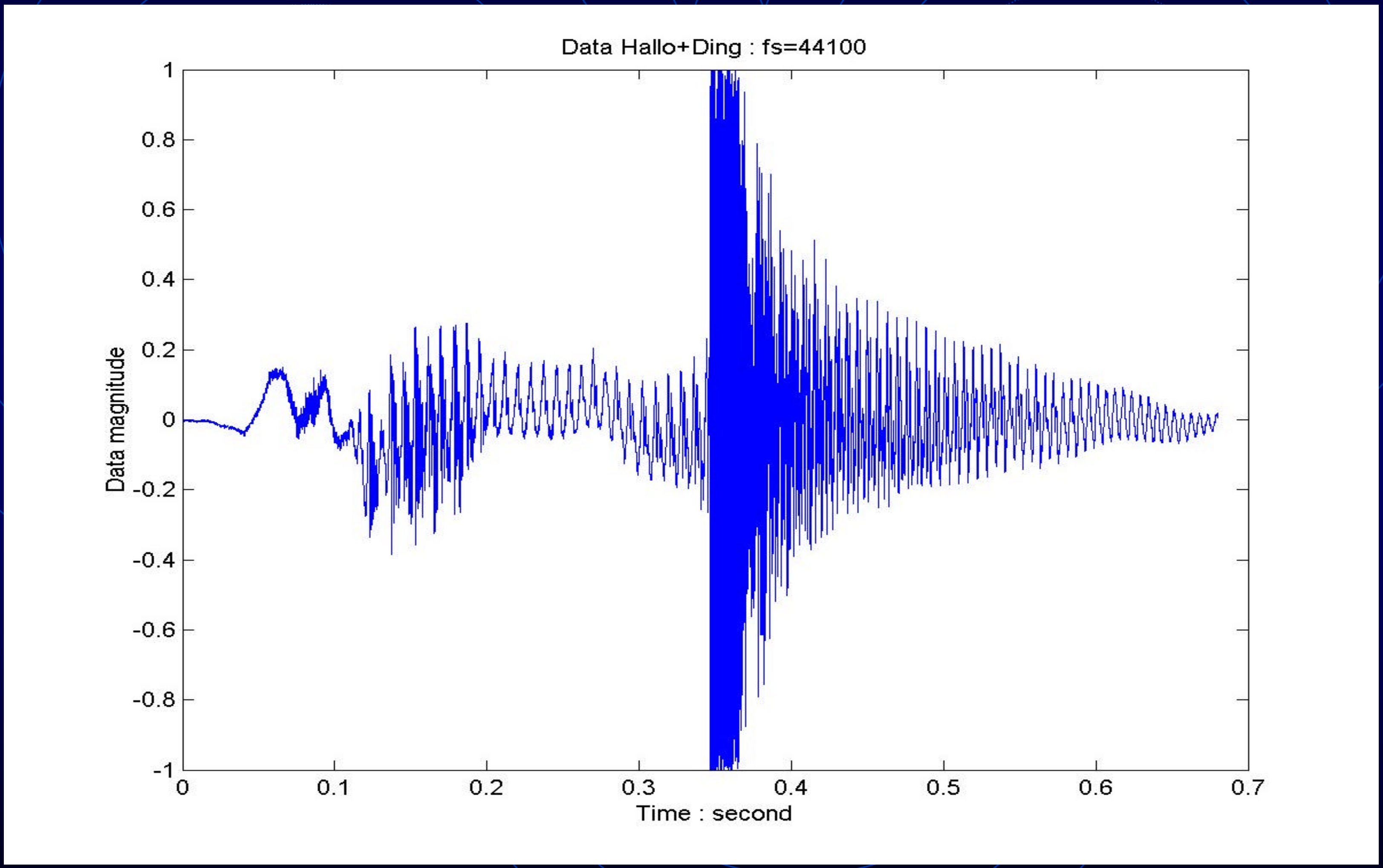
$$\text{Band Pass Filter:} \quad x_B(t) = \sum_{j=B}^{M} c_j \;.$$

An Example :
Removal of Unwanted Sound

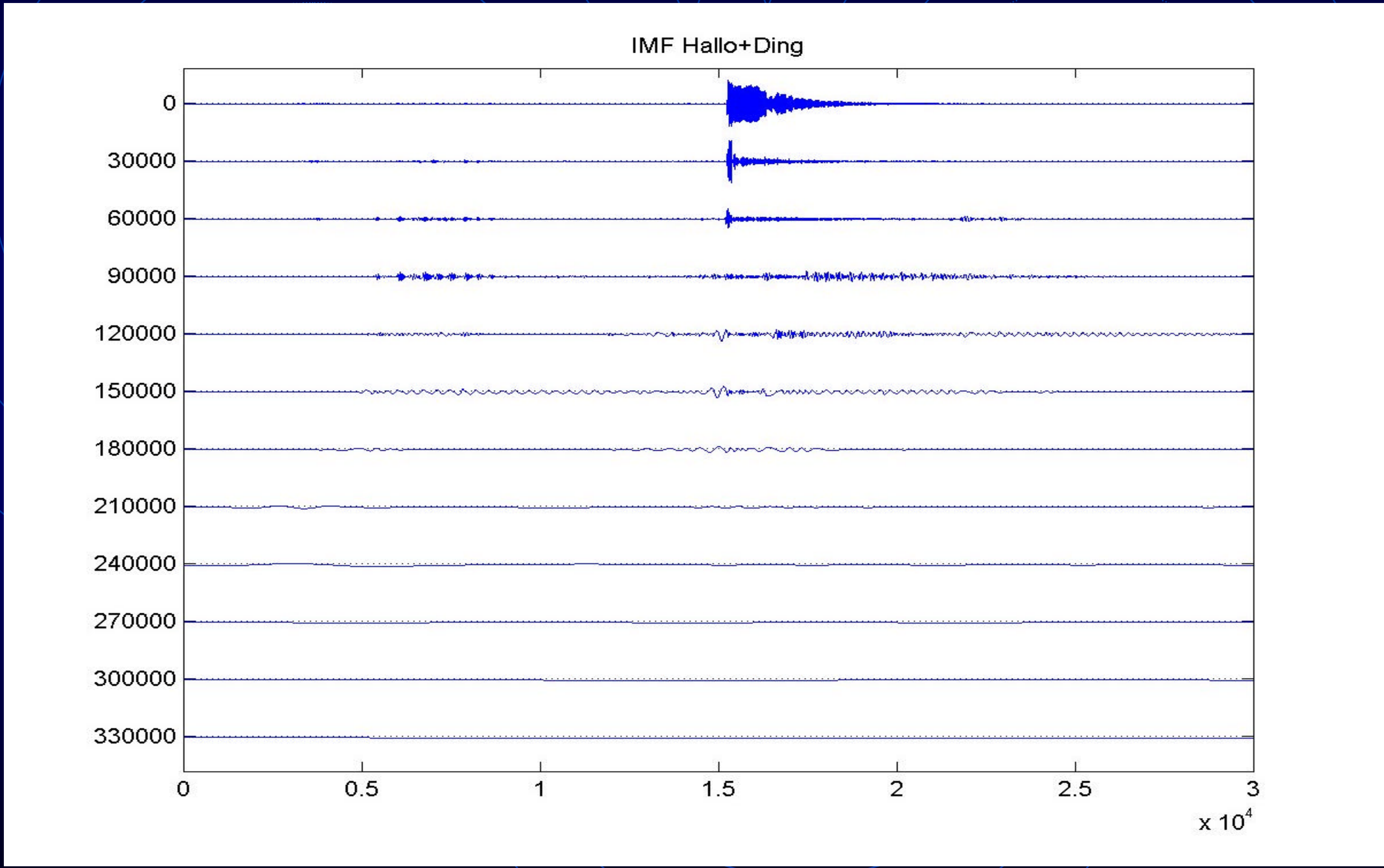
HHT Filtering to Separate

Ding from Hello

Data : Hallo + Ding



IMF : Hallo + Ding

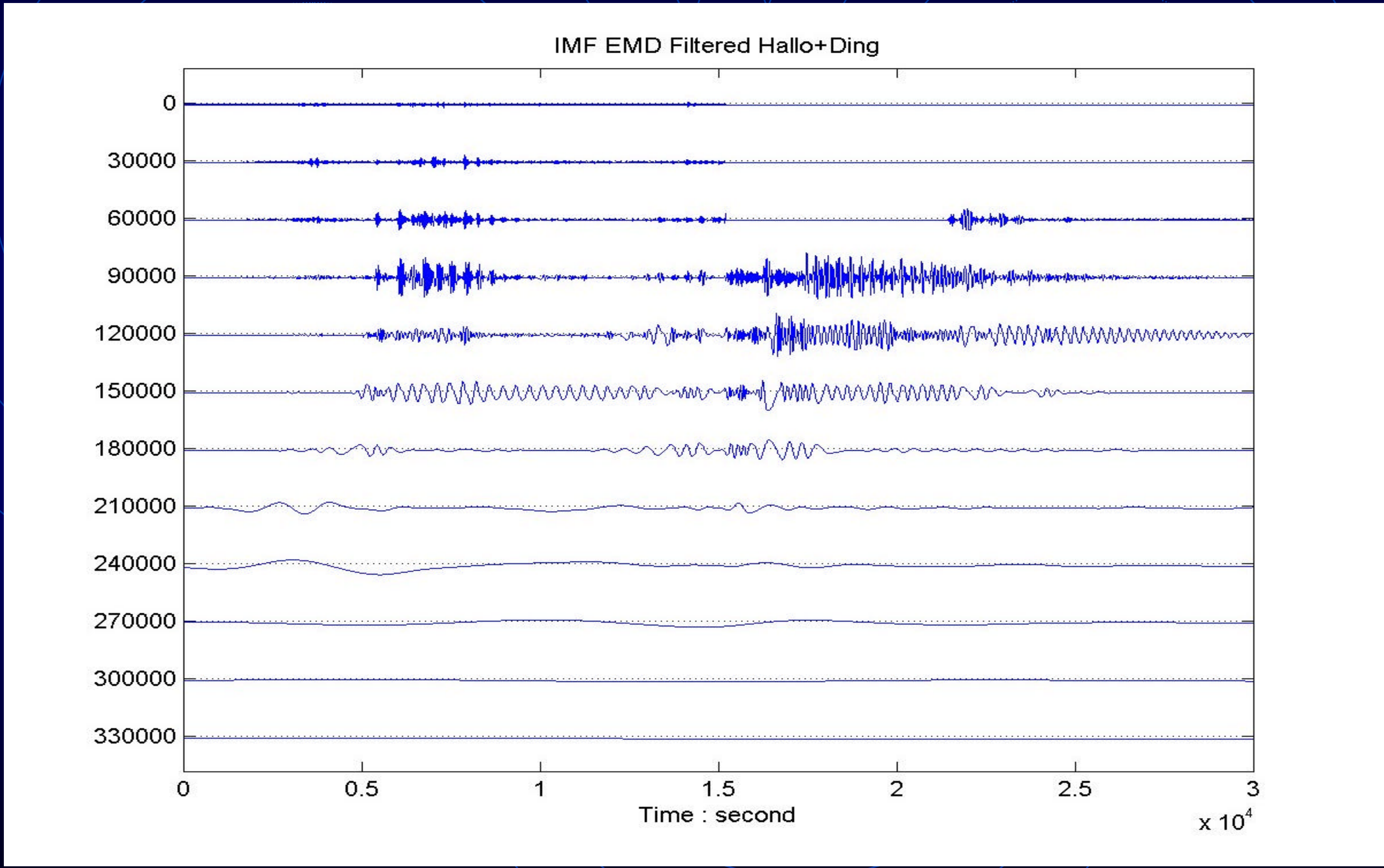


Filter for Hallo + Ding is defined as

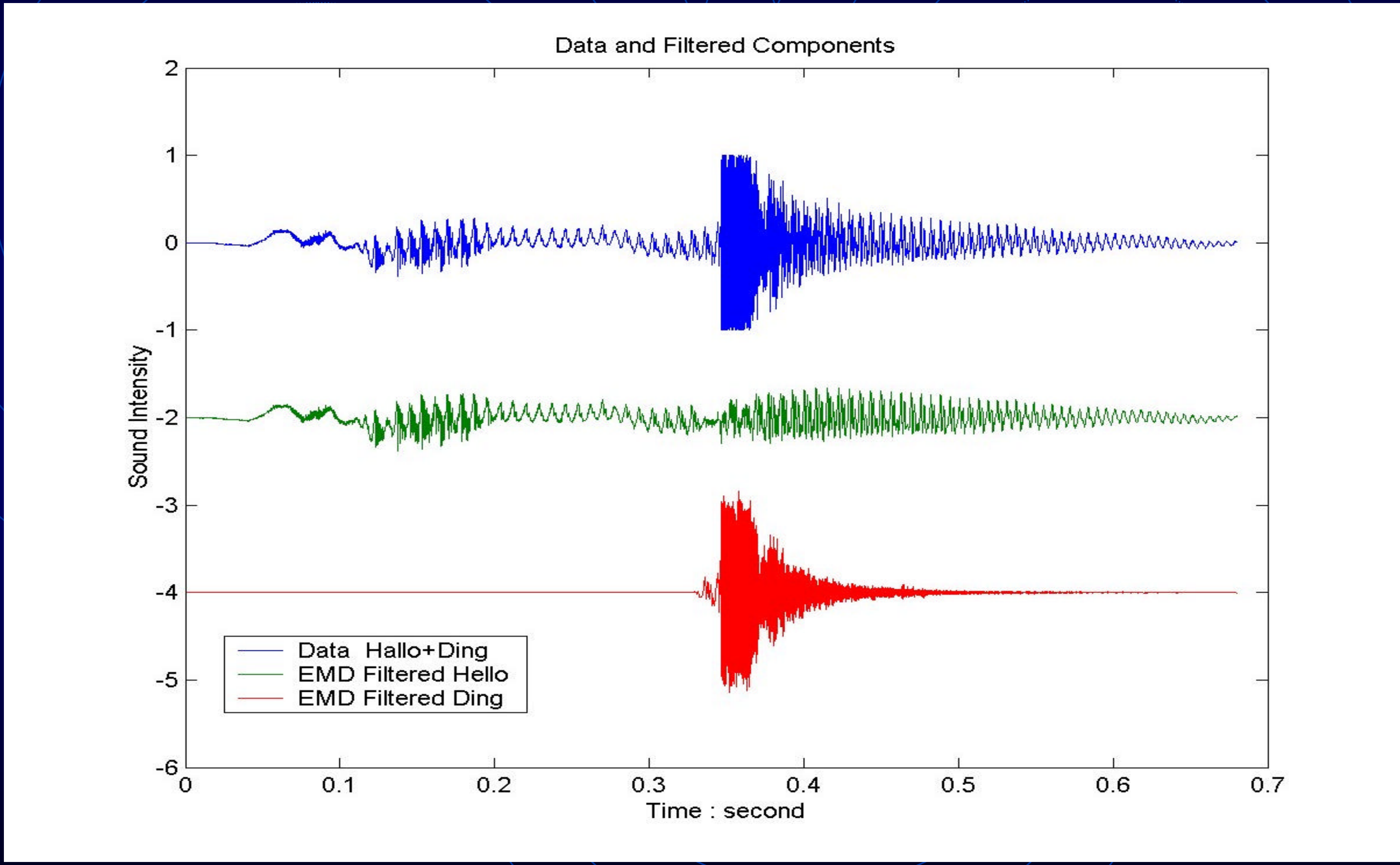
- c1(15200:30000) = 0;
- c2(15200:30000) = 0;
- c3(15200:21400) = 0;
- For c4 to c9 : q=[cos(2ð t/1200) +1]/2 ;

  for t=0:1200, centered at 15200.

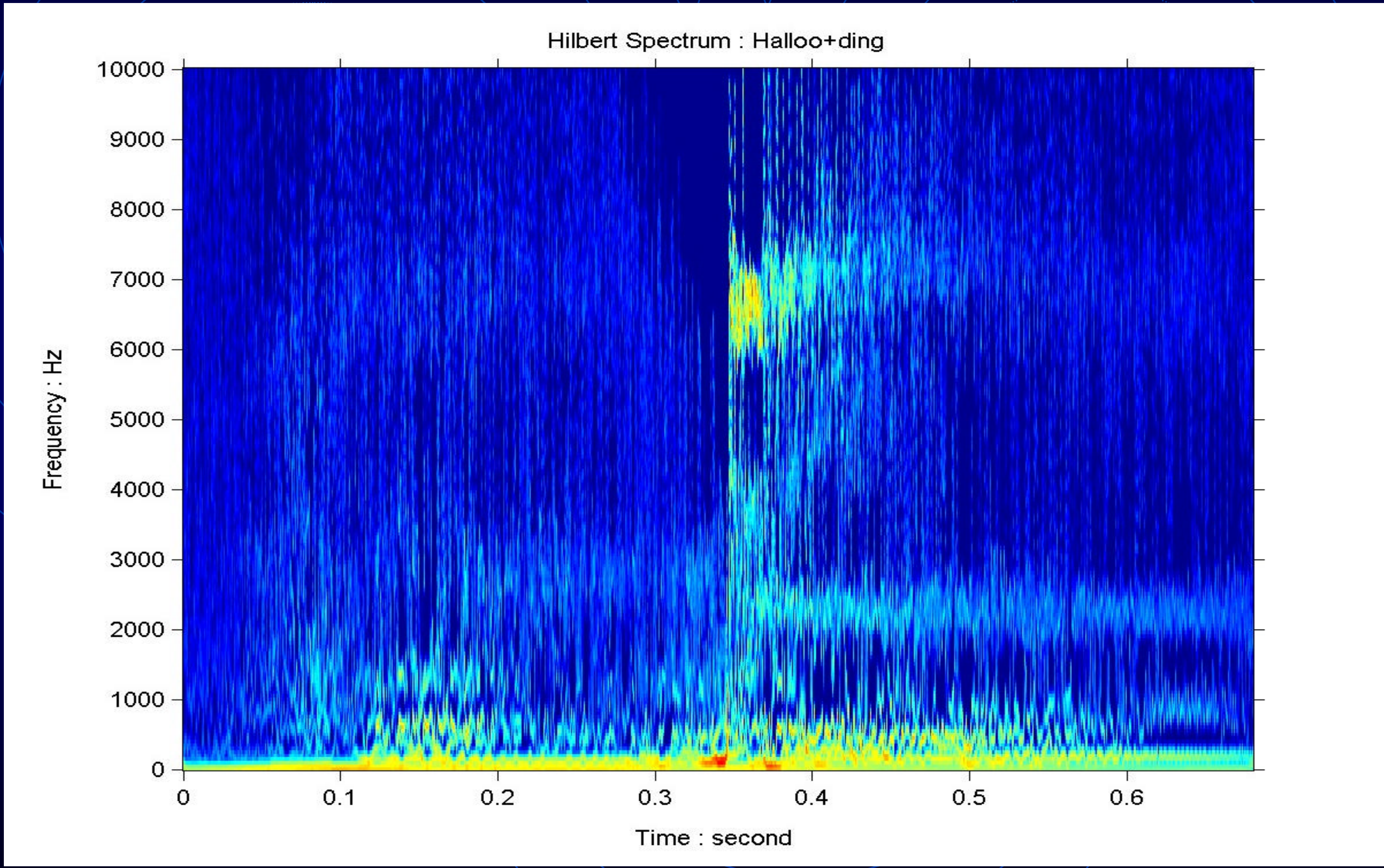
IMF Filtered : Hallo



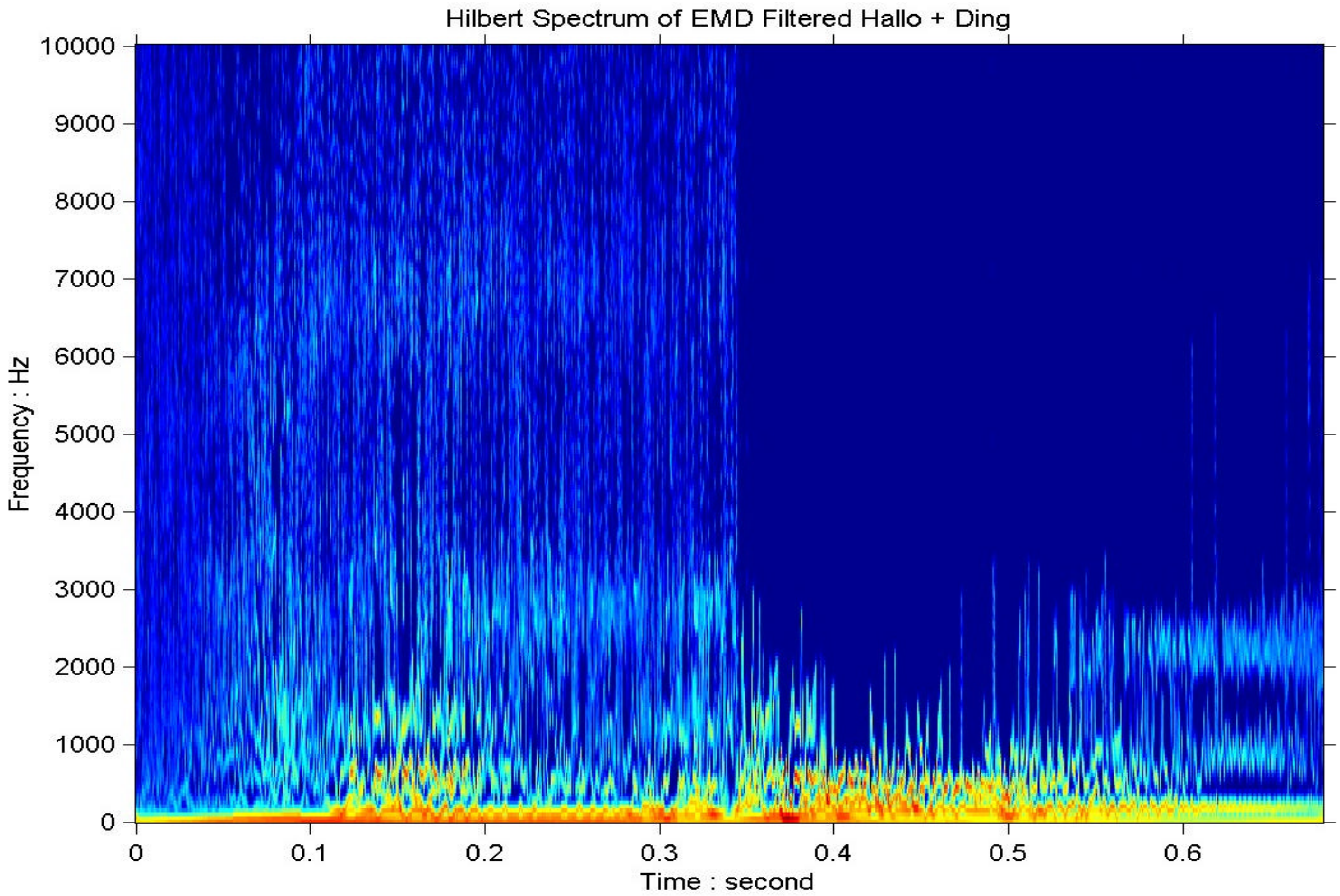
Data and Filtered Components



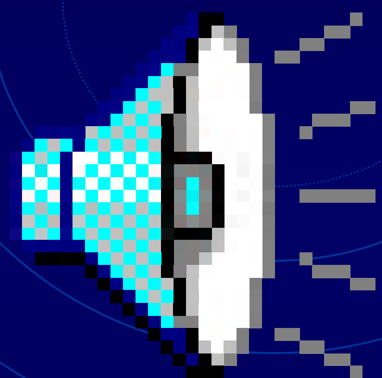
Hilbert Spectrum : Hallo + Ding



Hilbert Spectrum Filtered : Hallo

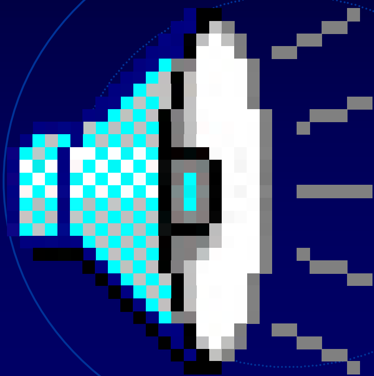


Sound Effects : Data

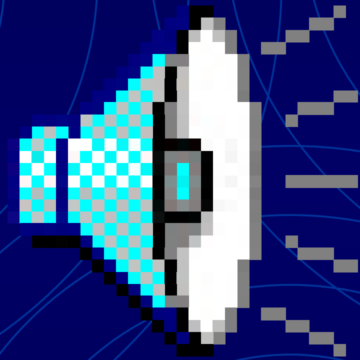


Raw data : Hallo + Ding

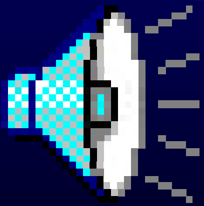
Sound Effects : Fourier Filtered



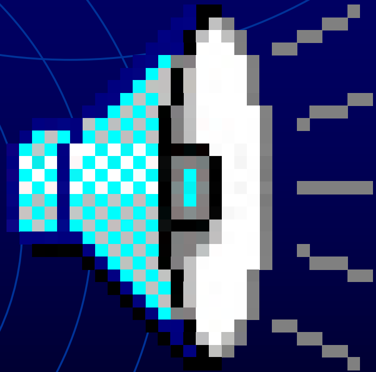
Fourier 3K



Fourier 2K

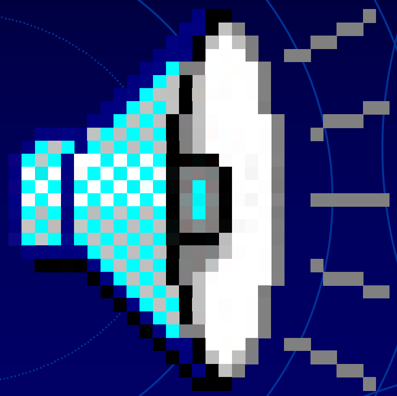


Original

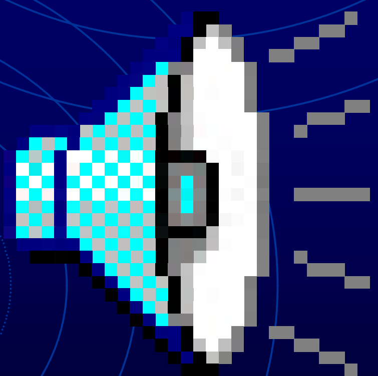


Fourier 1K

Sound Effects : EMD Filtered



EMD Filtered Hello



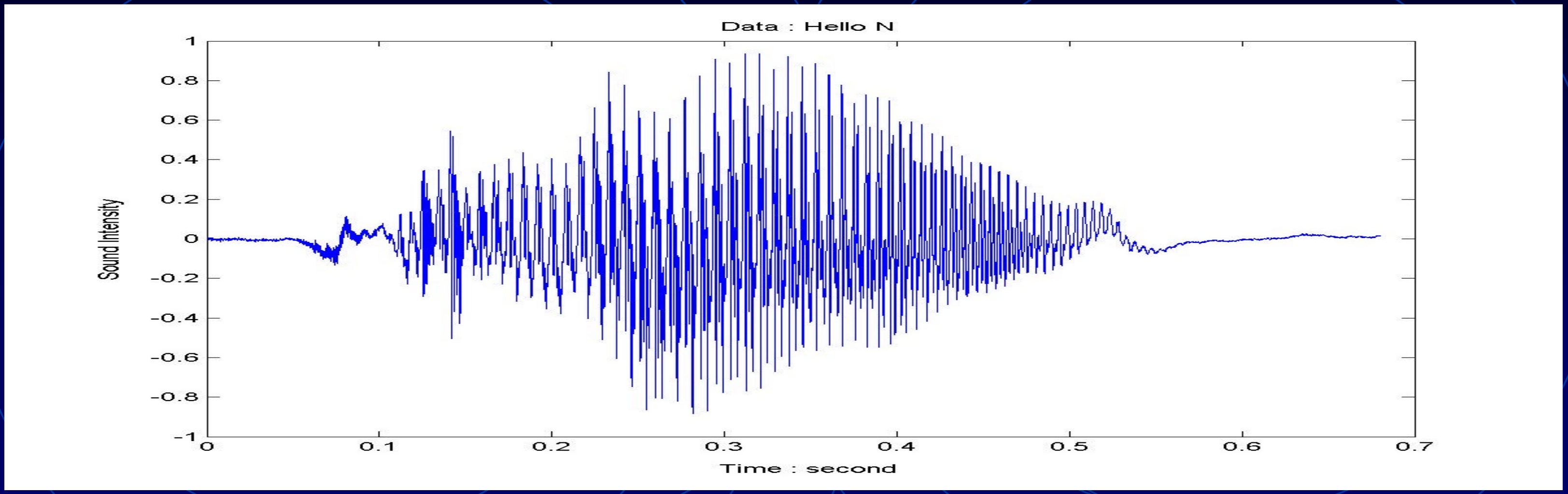
EMD Filtered Ding

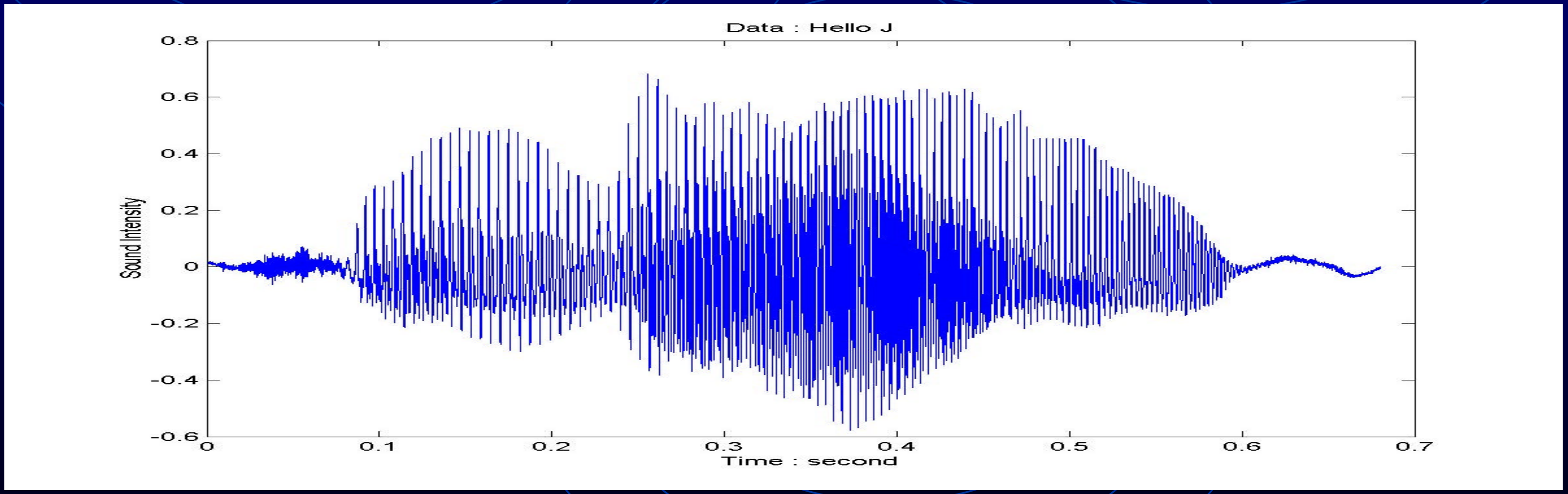
Original Sound

Hilbert Spectra for Different Speakers

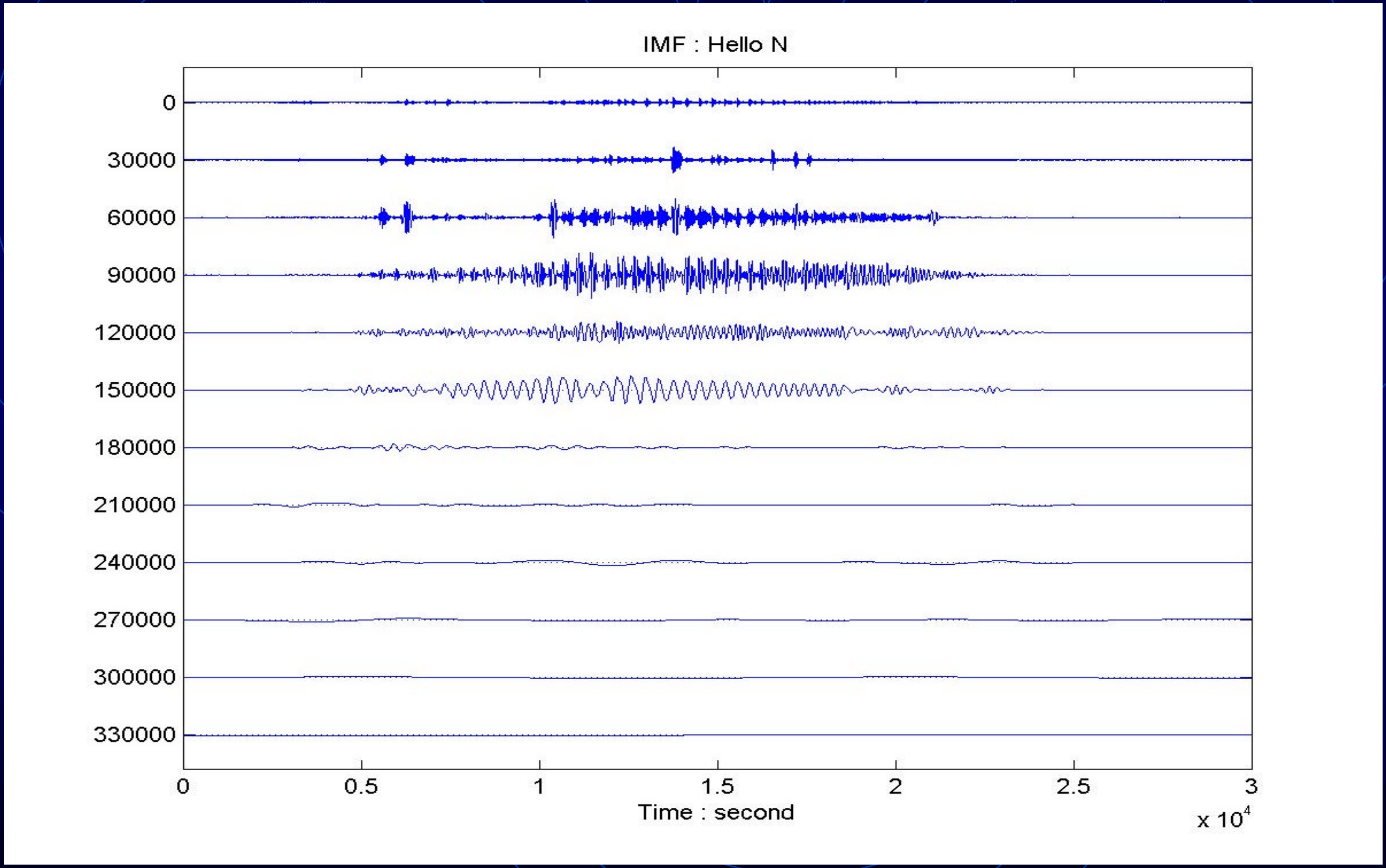
Potential Application for Speaker Identification

Difference between Speakers



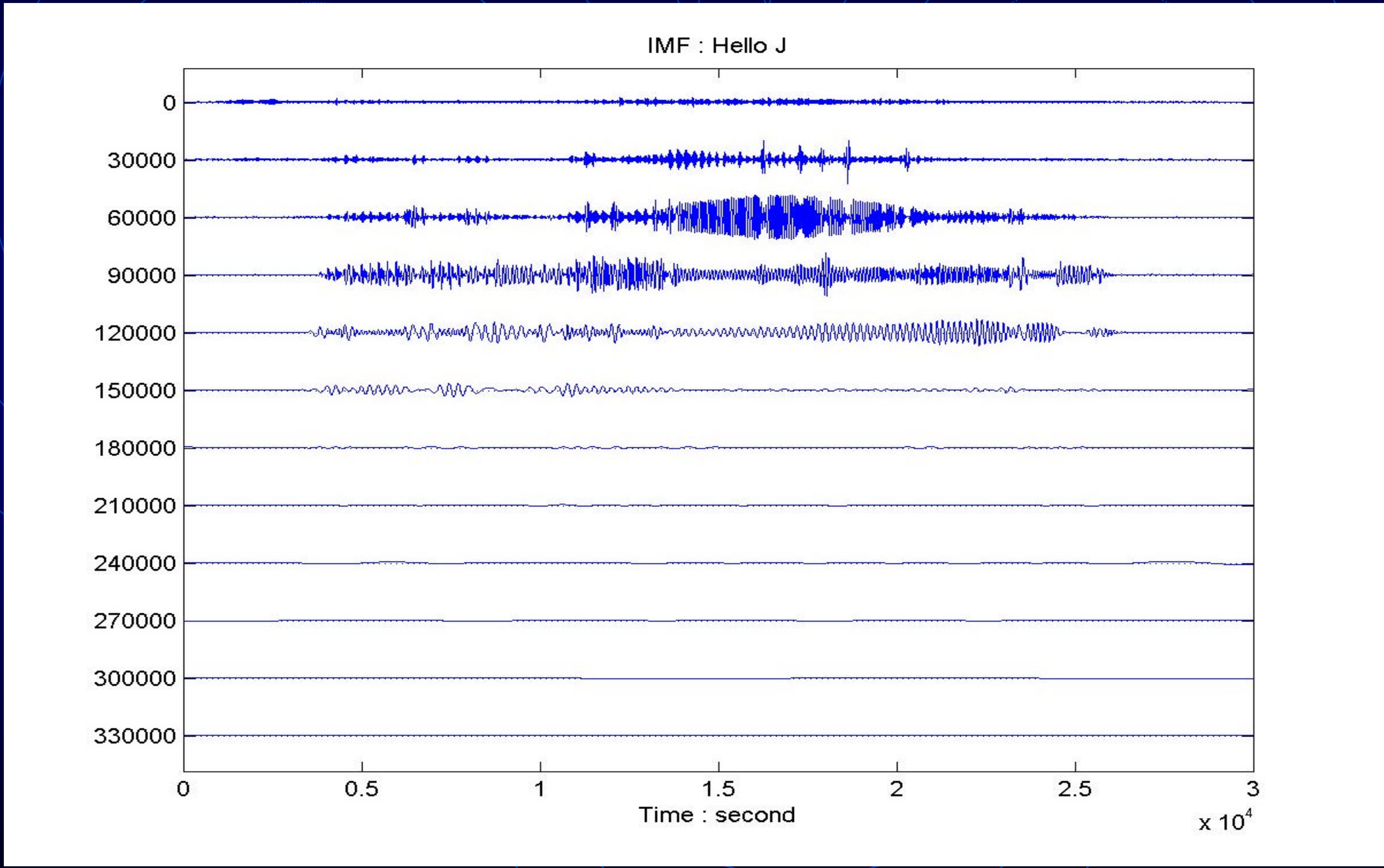


IMF : Hello N



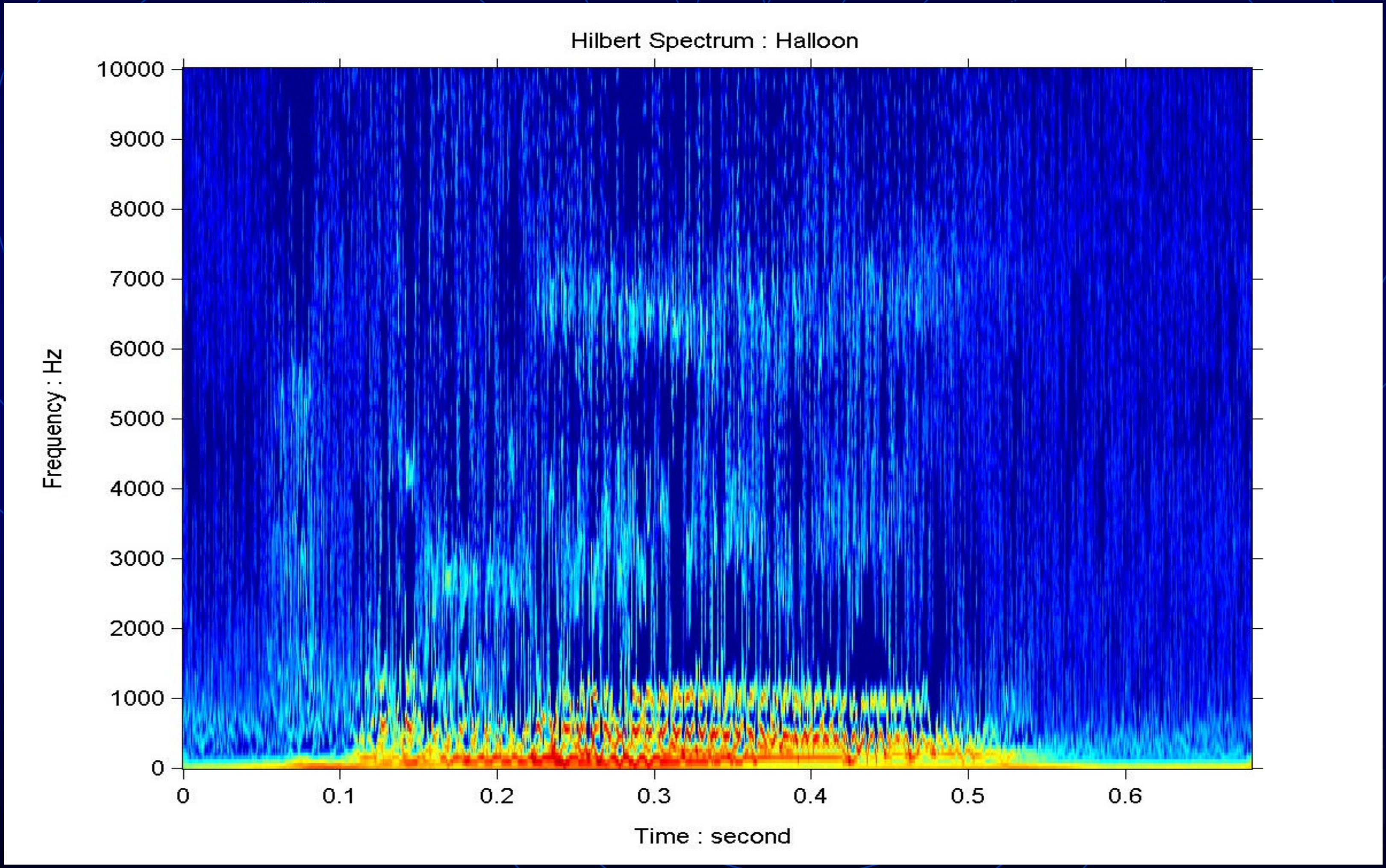
IMF : Hello N
0
30000
60000
90000
120000
150000
180000
210000
240000
270000
300000
330000
0
0.5
1
1.5
2
2.5
3
Time : second
x 10^4

IMF : Hello J

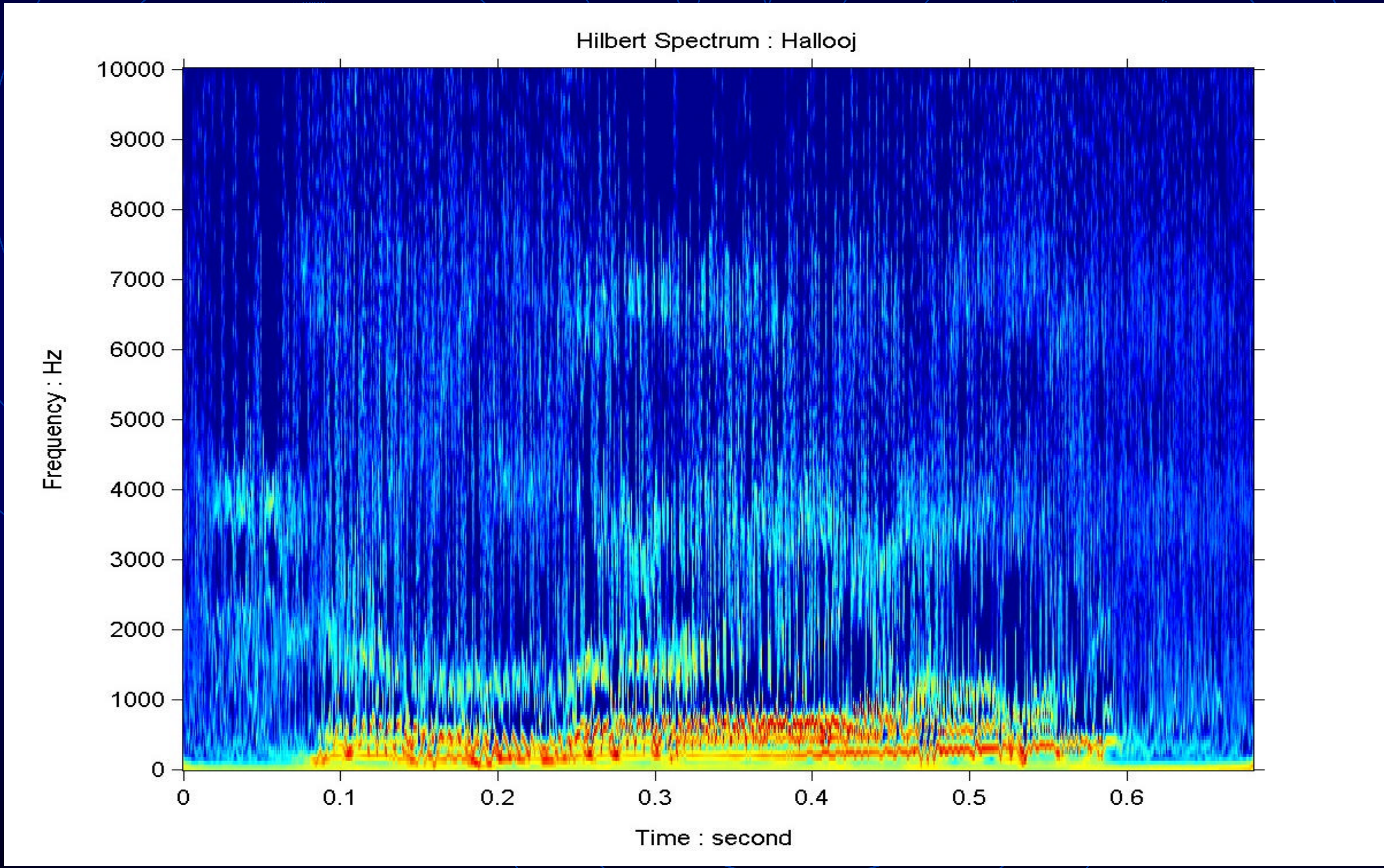


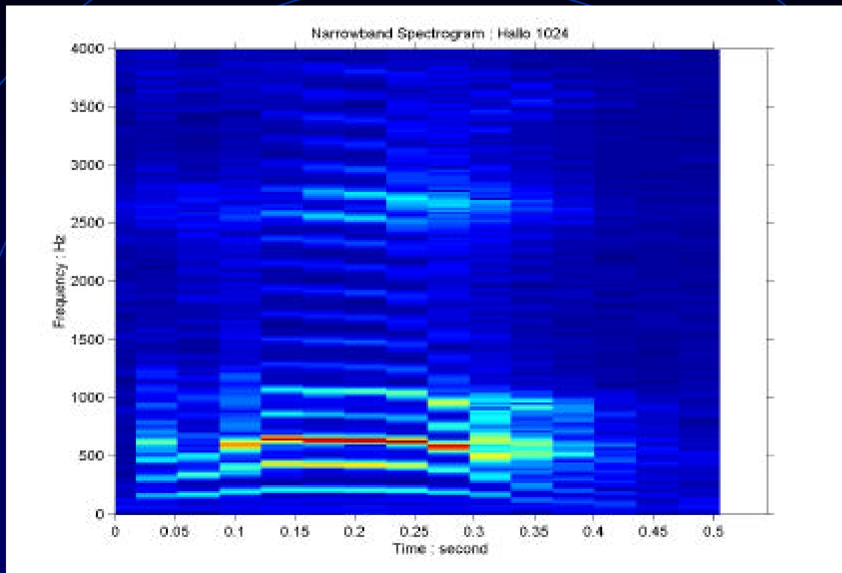
IMF : Hello J
0
30000
60000
90000
120000
150000
180000
210000
240000
270000
300000
330000
0
0.5
1
1.5
2
2.5
3
Time : second
x 10^4

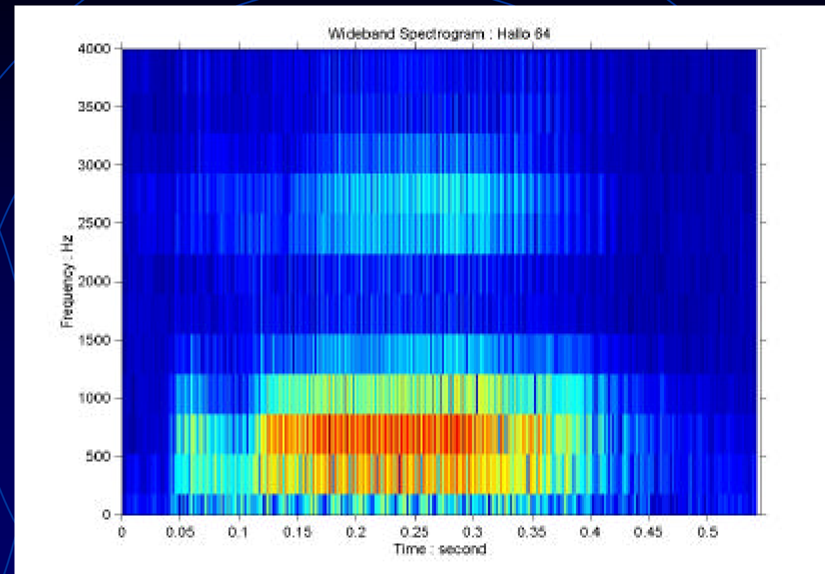
Hilbert Spectrum : Hello N

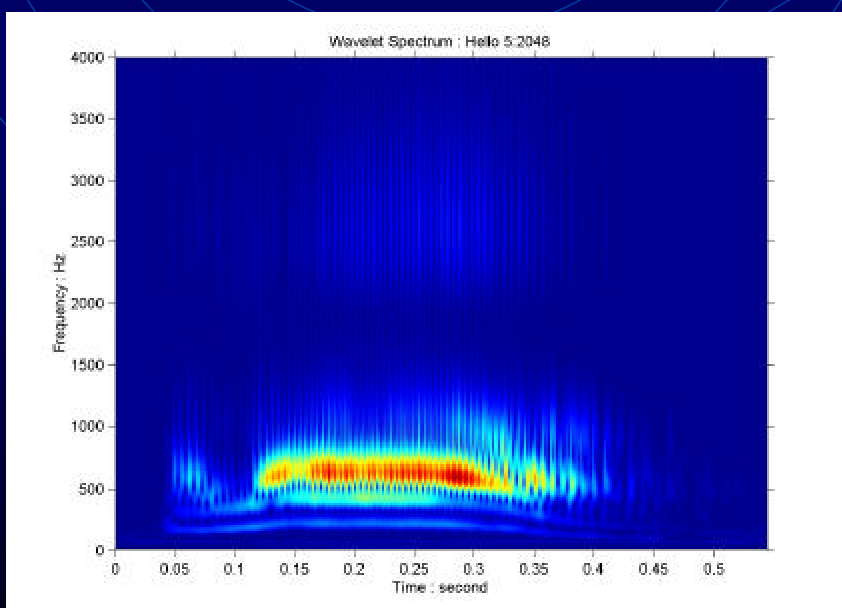


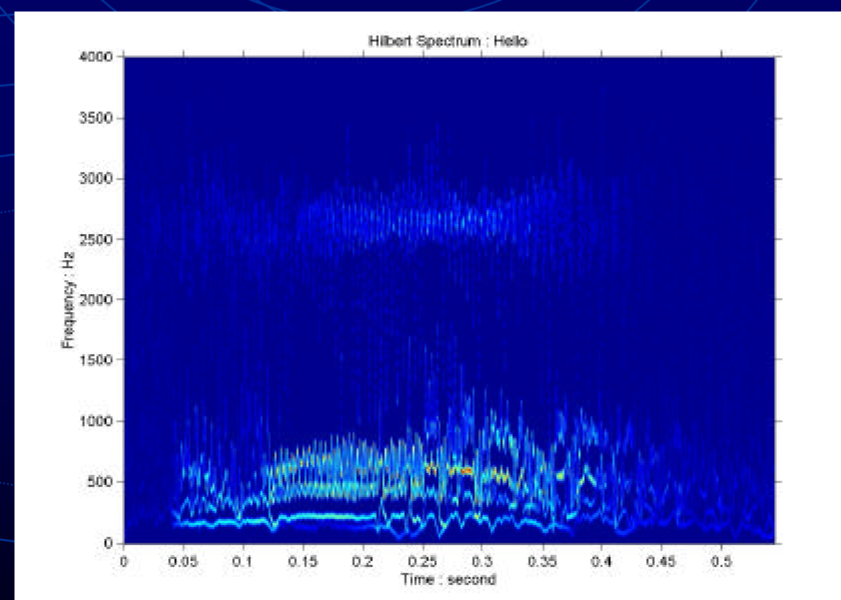
Hilbert Spectrum : Hello J



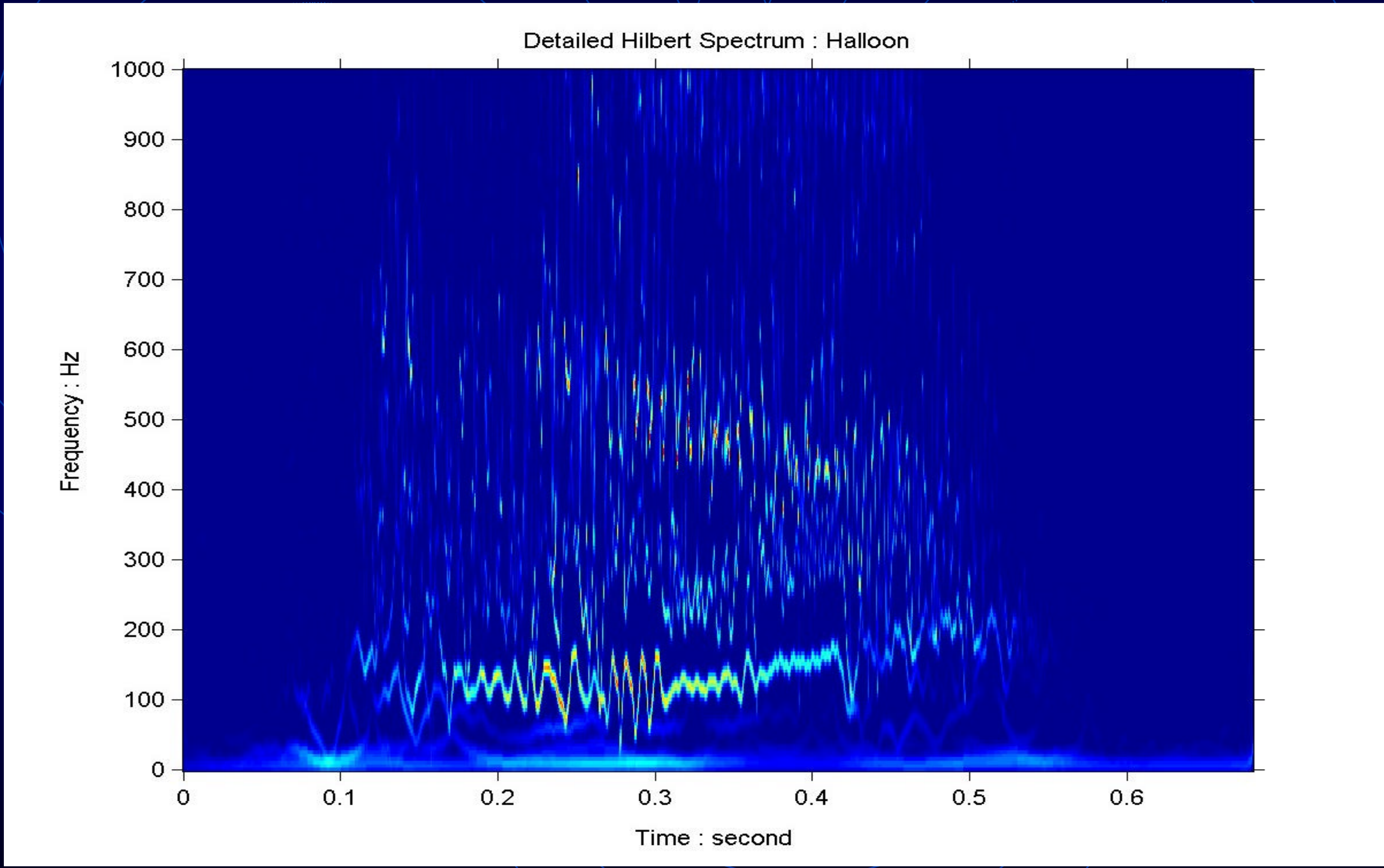

Narrowband Spectrogram : Hallo 1024
Frequency : Hz
Time : second


Wideband Spectrogram : Hallo 64
Frequency : Hz
Time : second

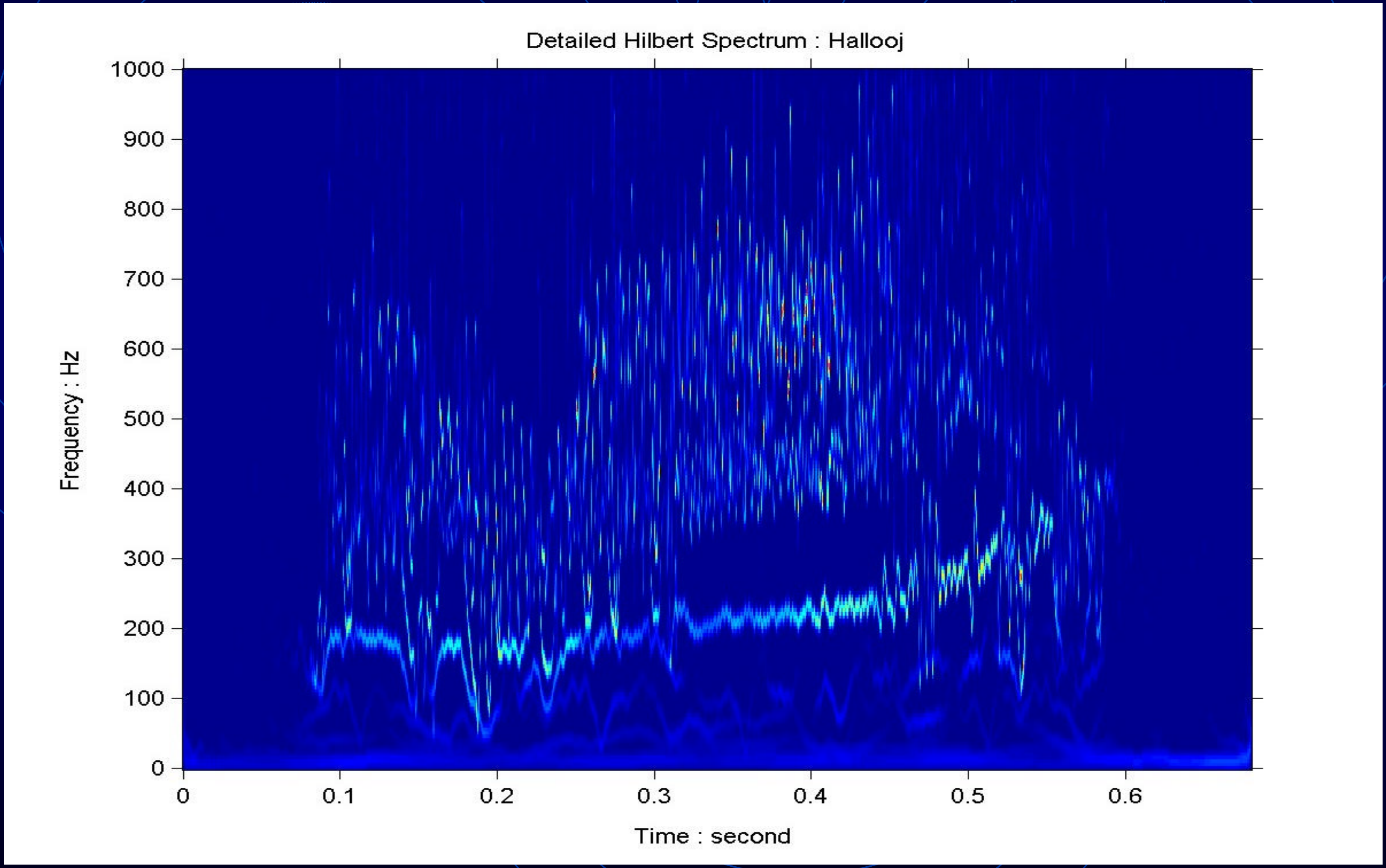

Wavelet Spectrum : Hello 5:2048
Frequency : Hz
Time : second


Hilbert Spectrum : Hello
Frequency : Hz
Time : second

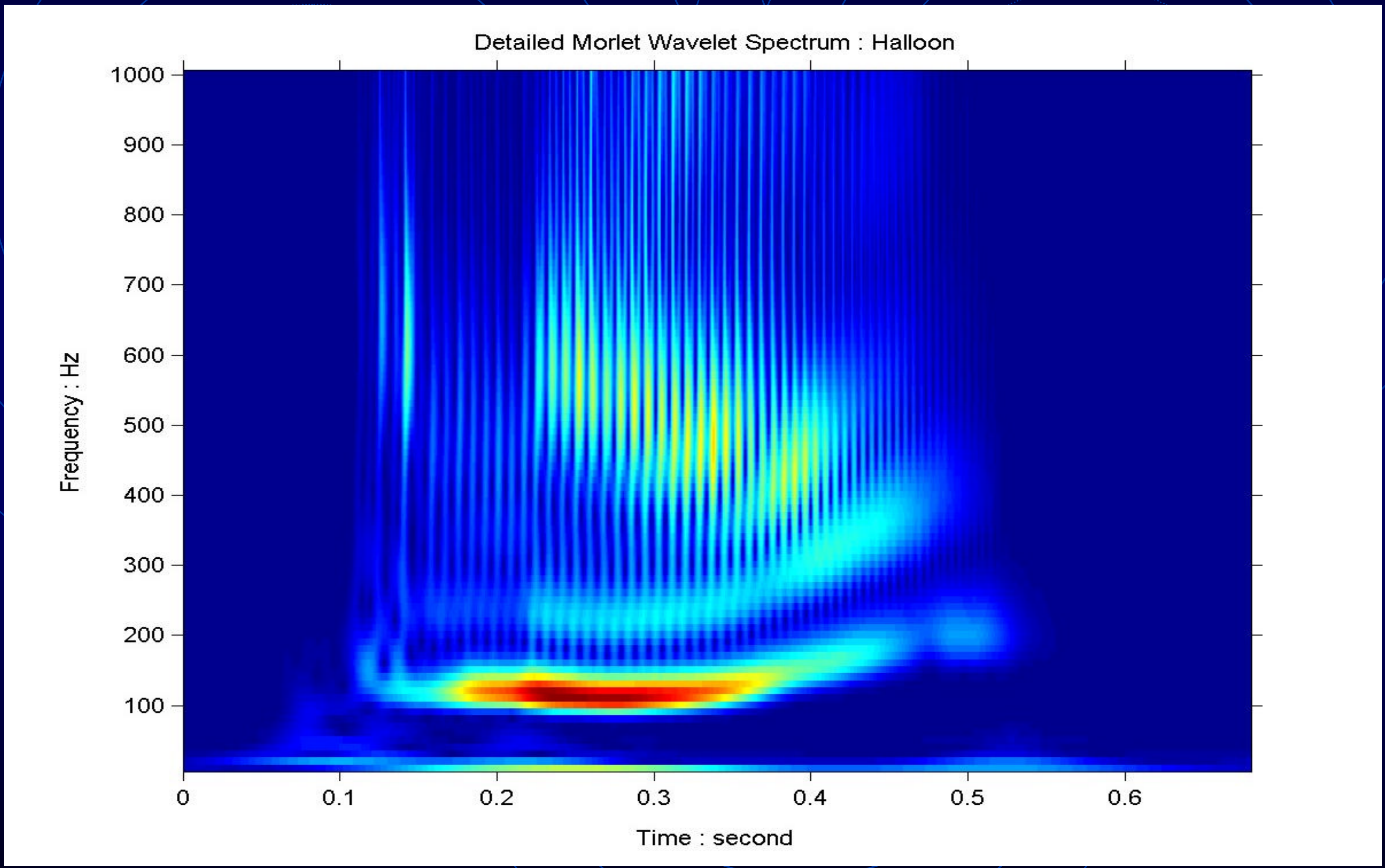
Detailed Hilbert Spectrum : Hello N



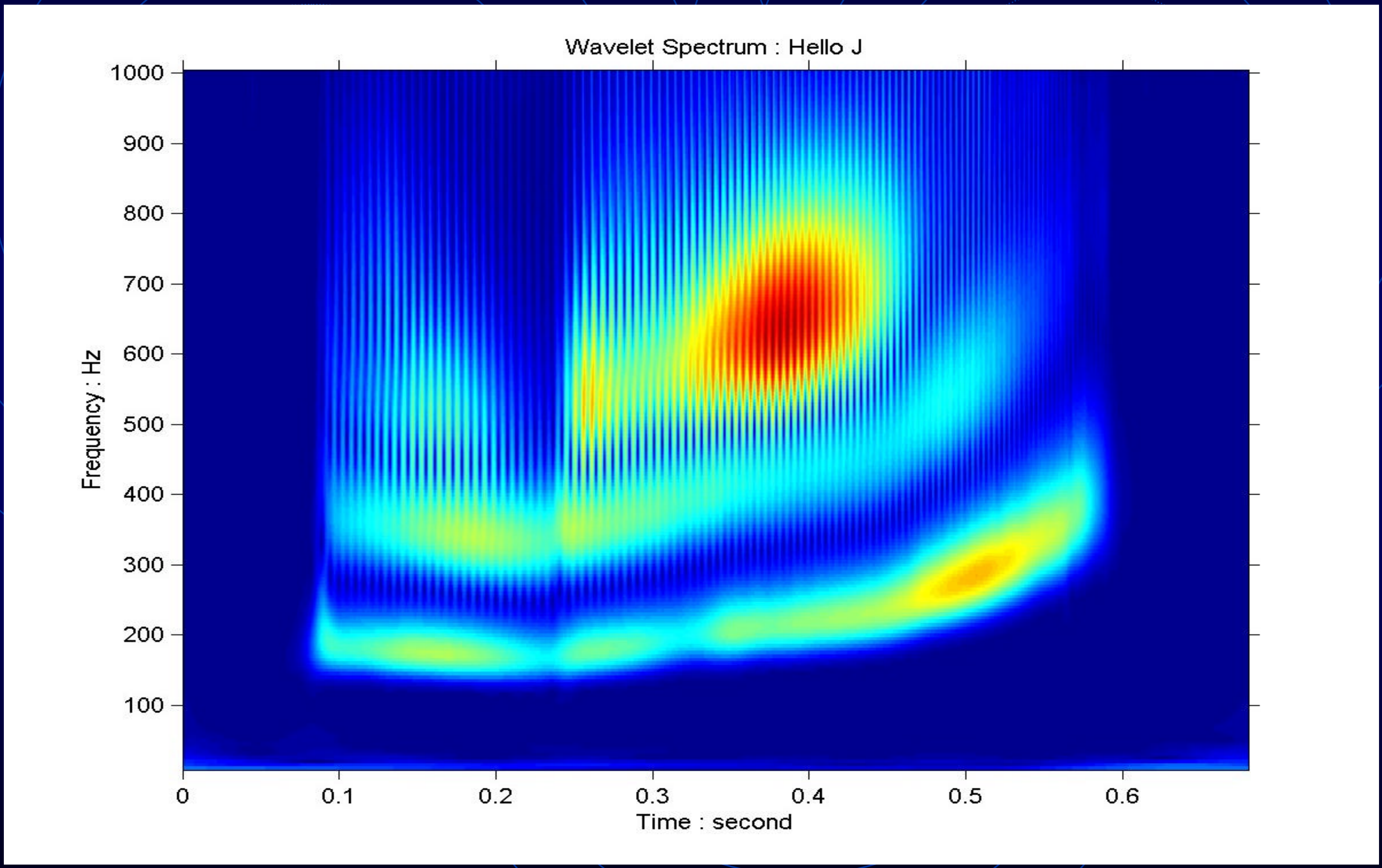
Detailed Hilbert Spectrum : Hello J



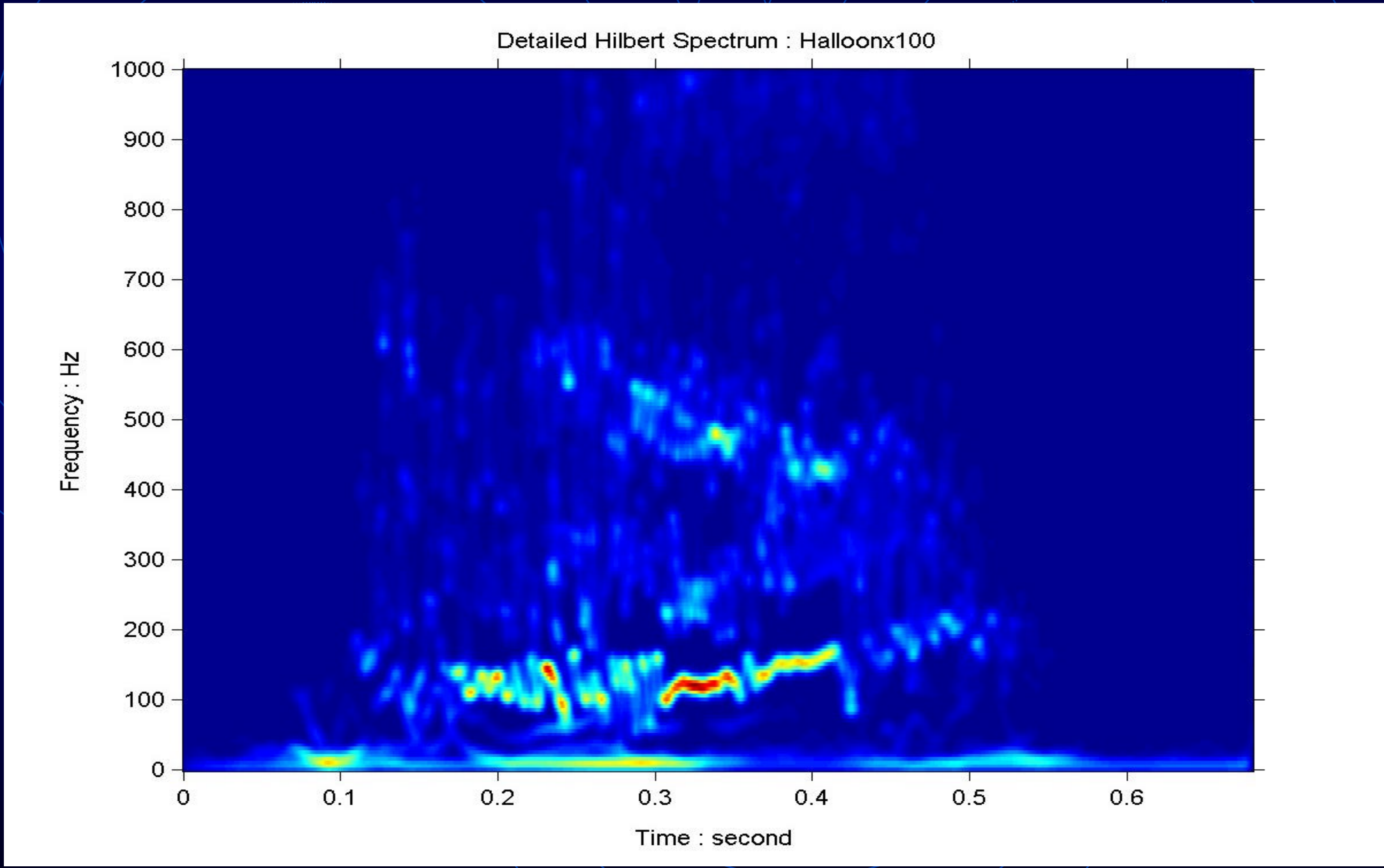
Detailed Wavelet Spectrum : Hello N



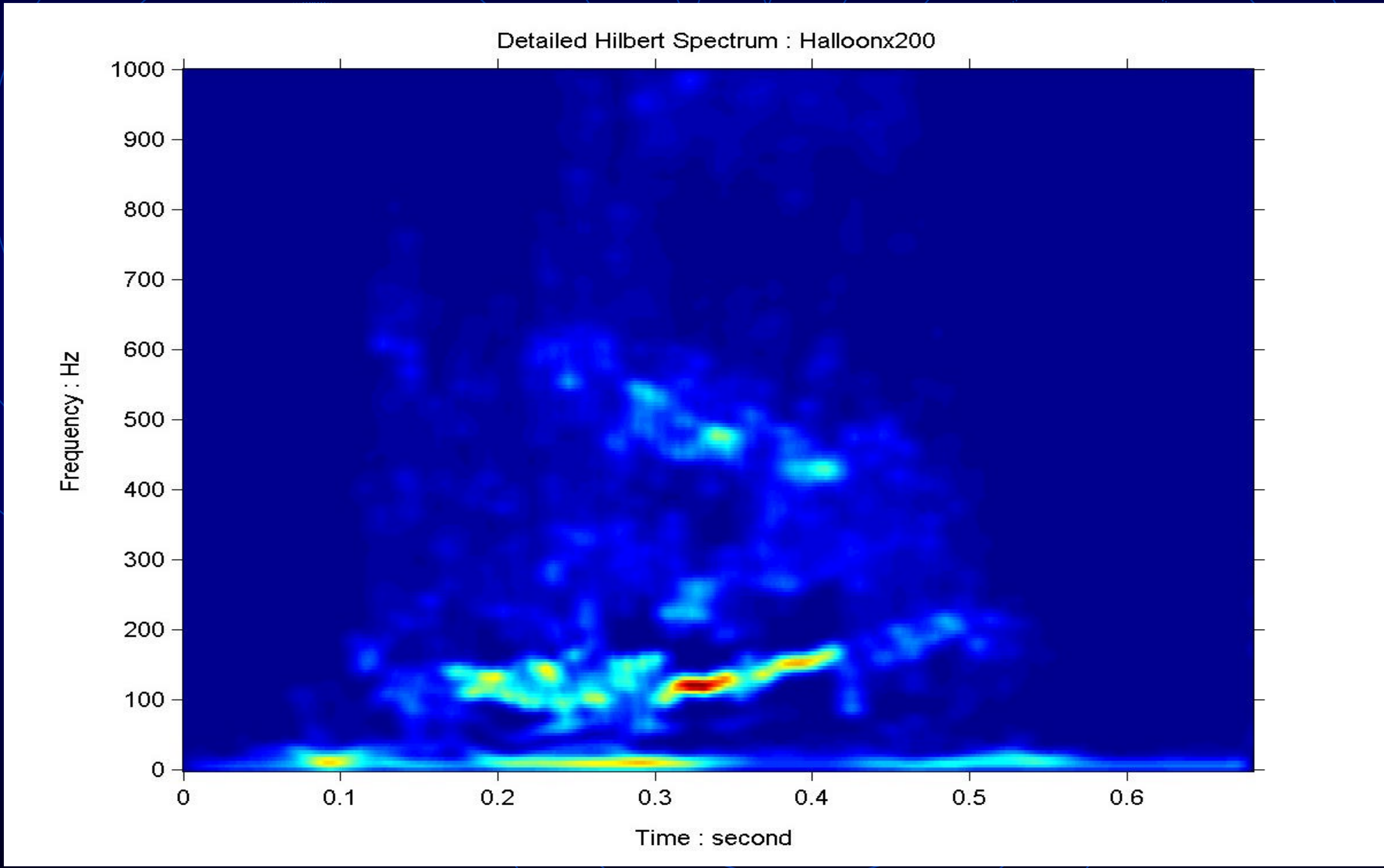
Detailed Wavelet Spectrum : Hello J



100 Smoothed H Spectrum : Hello N



200 Smoothed H Spectrum : Hello N

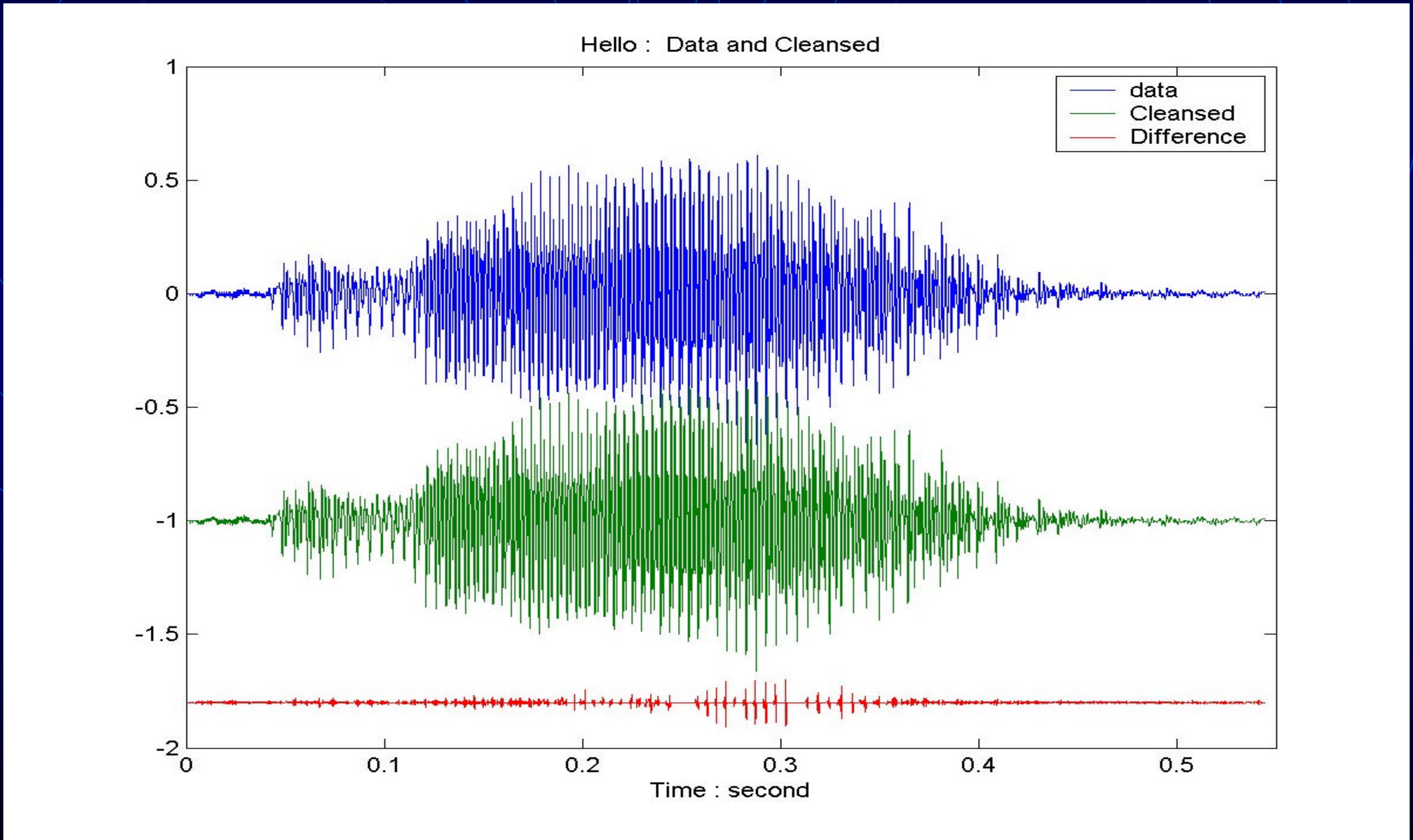


EMD as Filters :

The Effects of Harmonics

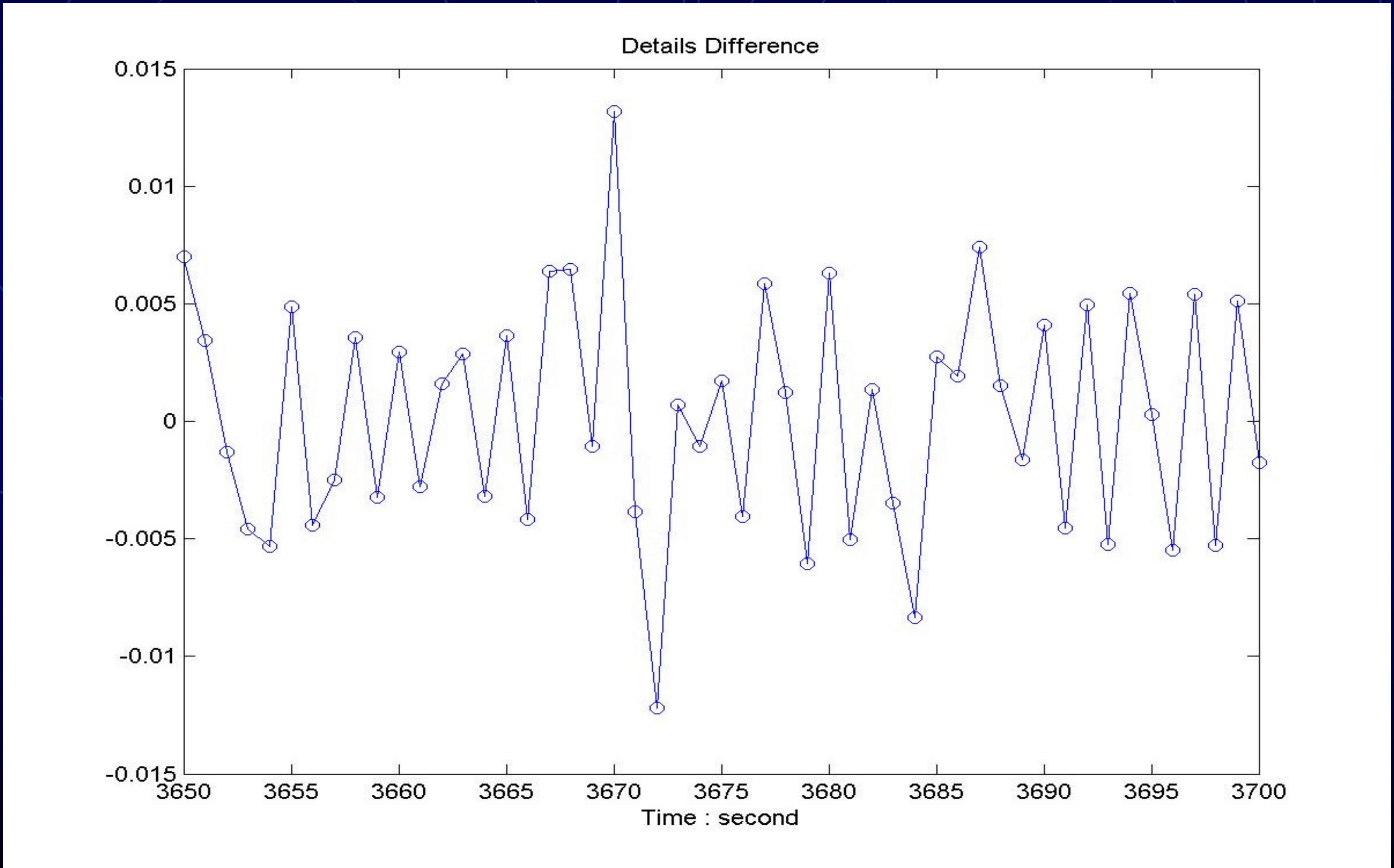
Speech Analysis :

Hello : The Effects of Harmonics and EMD filtering



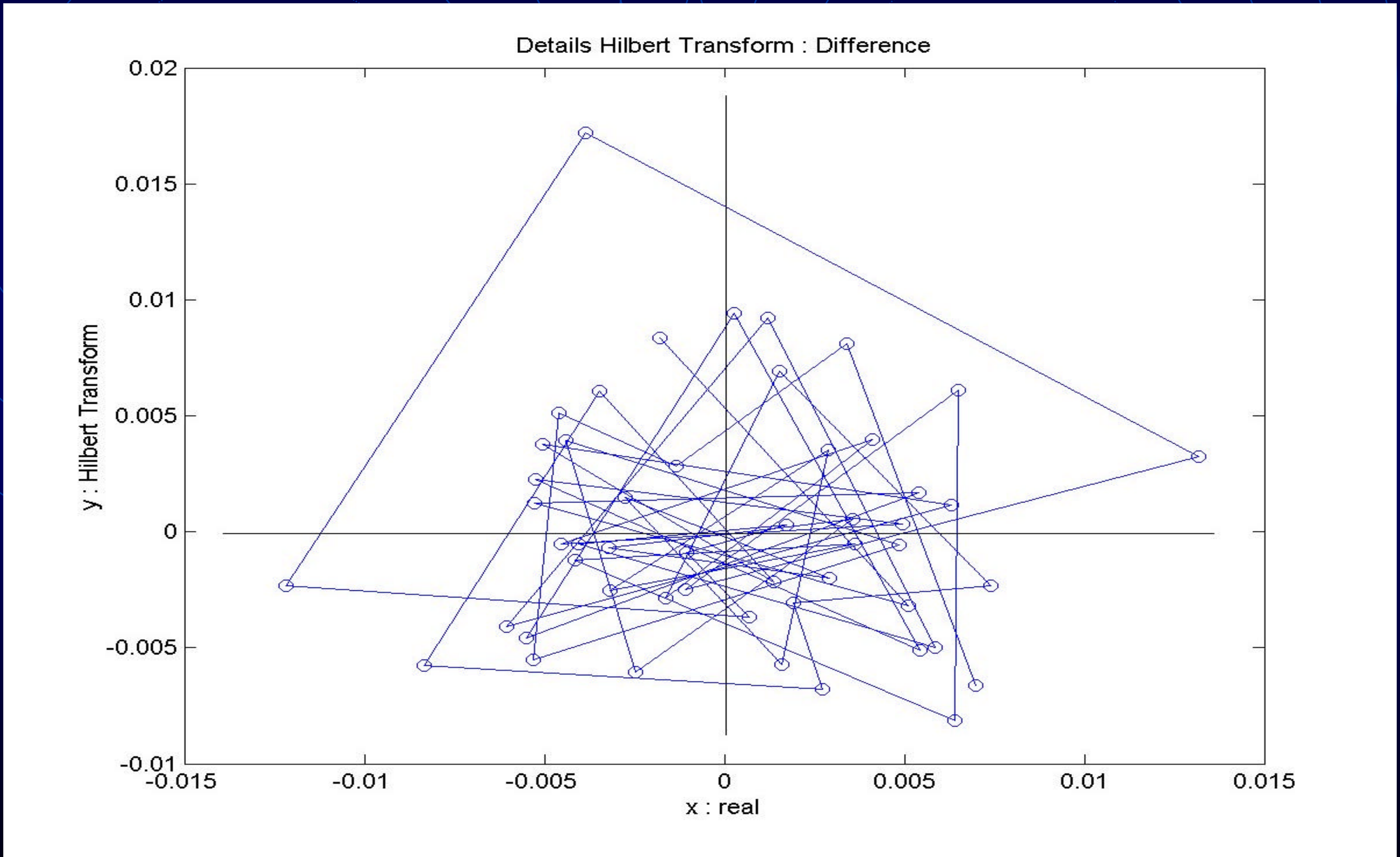
Speech Analysis :

Hello : Details of the Difference Data



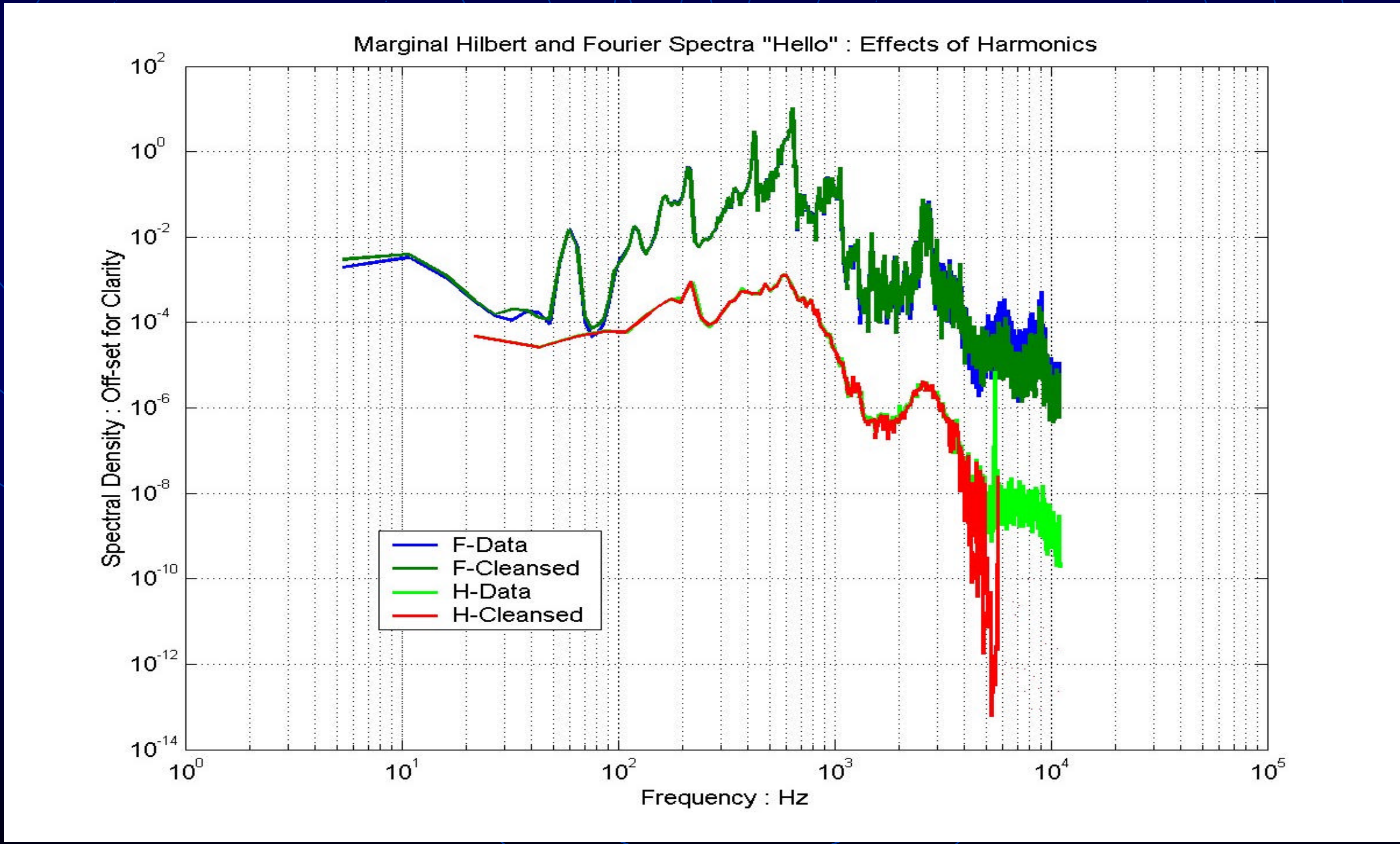
Speech Analysis :

Hello : The Hilbert Transform of the Difference Data



Speech Analysis :

Hello : The Effects of Harmonics and EMD filtering



Summary

- **Numerous application possibilities**
- **Intellectual property protected**
- **Concepts demonstrated in many applications**
- **Licensing and partnering opportunity**
- **Enabling technology with significant commercial potential**
- **Significant benefits**
  - Precision, flexibility, accuracy, easy implementation,

Contact Info

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